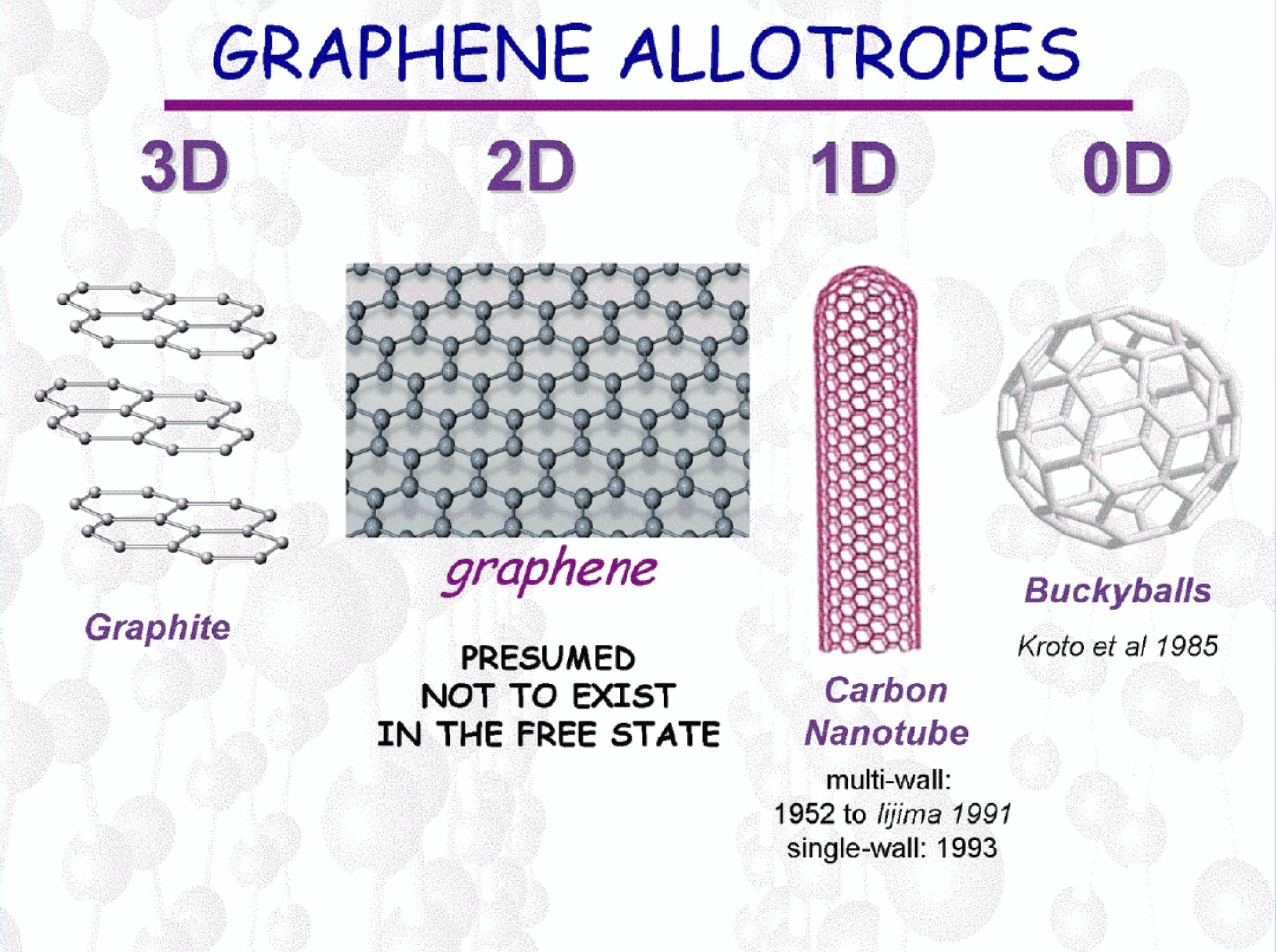
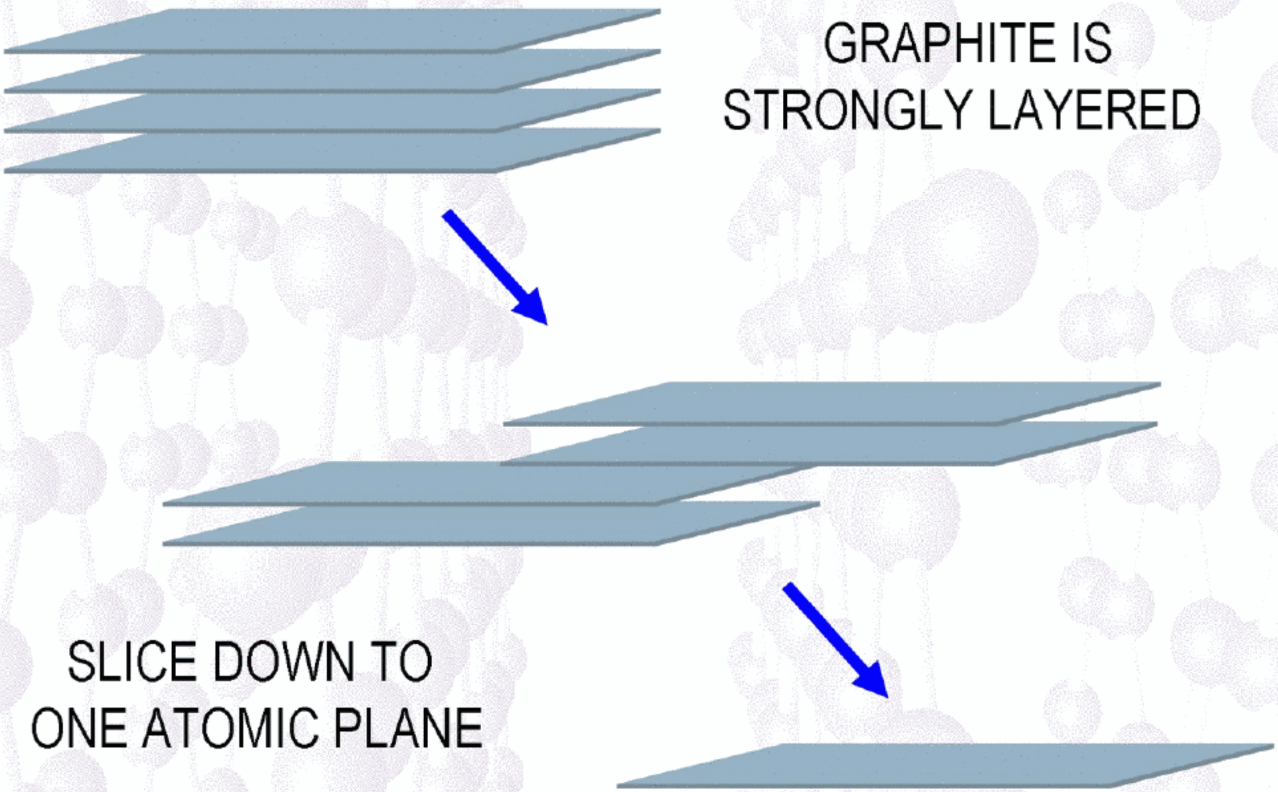
Graphene and Dirac fermions

M. Riccò


GRAPHENE ALLOTROPES
3D
2D
1D
0D
Graphite
graphene
PRESUMED NOT TO EXIST IN THE FREE STATE
Carbon Nanotube
multi-wall: 1952 to Iijima 1991
single-wall: 1993
Buckyballs
Kroto et al 1985

Extracting a Single Plane



individual atomic sheets: do they exist?

PHYSICAL REVIEW VOLUME 176, NUMBER 1 5 DECEMBER 1968

Crystalline Order in Two Dimensions*

N. D. Mermin†
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York
(Received 1 July 1968)

If N classical particles in two dimensions interacting through a pair potential $\Phi(\vec{r})$ are in equilibrium in a parallelogram box, it is proved that every $\vec{k} \neq 0$ Fourier component of the density must vanish in the thermodynamic limit, provided that $\Phi(\vec{r}) - \lambda r^2 |\nabla^2 \Phi(\vec{r})|$ is integrable at $r = \infty$ and positive and nonintegrable at $r = 0$, both for $\lambda = 0$ and for some positive λ. This result excludes conventional crystalline long-range order in two dimensions for power-law potentials of the Lennard-Jones type, but is inconclusive for hard-core potentials. The corresponding analysis for the quantum case is outlined. Similar results hold in one dimension.

Produzione del grafene

Scissione micromeccanica della HOPG

(A.Geim & K. Novoselov, Manchester University 2004)

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,[1] A. K. Geim,[1*] S. V. Morozov,[2] D. Jiang,[1] Y. Zhang,[1] S. V. Dubonos,[2] I. V. Grigorieva,[1] A. A. Firsov[2]

We describe monocrystalline graphitic films, which are a few atoms thick but are nonetheless stable under ambient conditions, metallic, and of remarkably high quality. The films are found to be a two-dimensional semimetal with a tiny overlap between valence and conductance bands, and they exhibit a strong ambipolar electric field effect such that electrons and holes in concentrations up to 10^{13} per square centimeter and with room-temperature mobilities of ~10,000 square centimeters per volt-second can be induced by applying gate voltage.

The ability to control electronic properties of a material by externally applied voltage is at the heart of modern electronics. In many cases, it is the electric field effect that allows one to vary the carrier concentration in a semiconductor device and, consequently, change an electric current through it. As the semiconductor industry is nearing the limits of performance improvements for the current technologies dominated by silicon, there is a constant search for new, nontraditional materials whose properties can be controlled by the electric field. The most notable recent examples of such materials are organic conductors (*1*) and carbon nanotubes (*2*). It has long been tempting to extend the use of the field effect to metals [e.g., to develop all-metallic transistors that could be scaled down to much smaller sizes and would consume less energy and operate at higher frequencies

[1]Department of Physics, University of Manchester, Manchester M13 9PL, UK. [2]Institute for Microelectronics Technology, 142432 Chernogolovka, Russia.

*To whom correspondence should be addressed. E-mail: geim@man.ac.uk

Structural stability

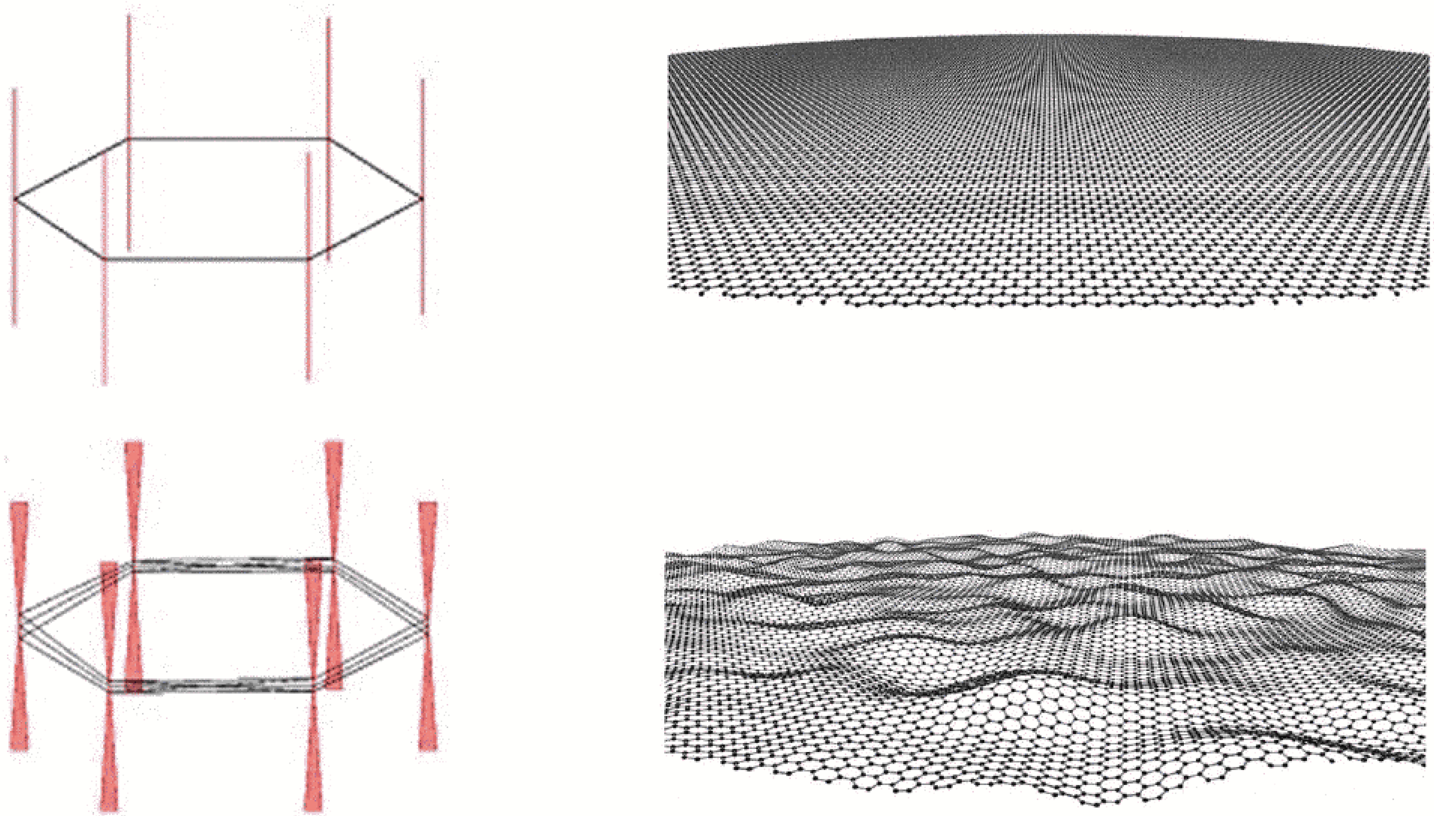
Mermin-Wagner theorem→ No long range order in 2D

Dynamics → divergence of the larger wavelenght phonons in 2D

Lindemann criterion <u> ~ 0.1d fusion

Anharmonic coupling terms between bending and stretching modes suppress these destructive fluctuations.

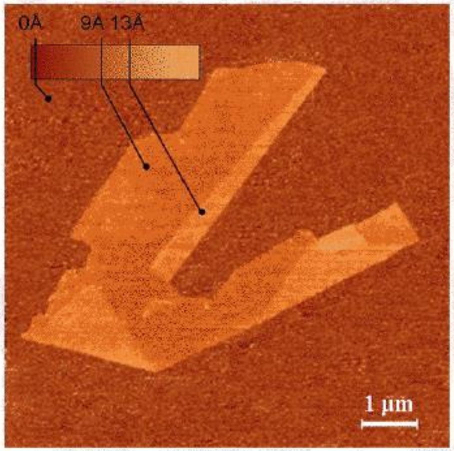
Graphene shows “ripples”



cones in Fourier space rather than rods

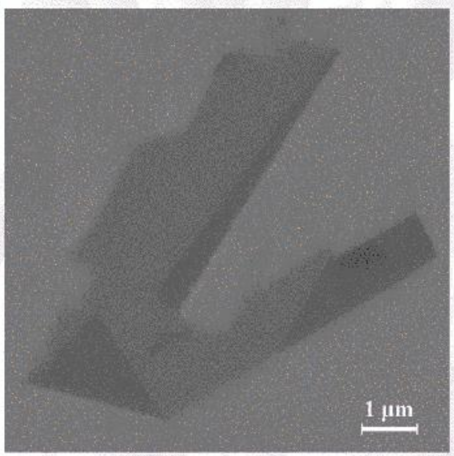
Free-Standing Graphene

AFM



low throughput

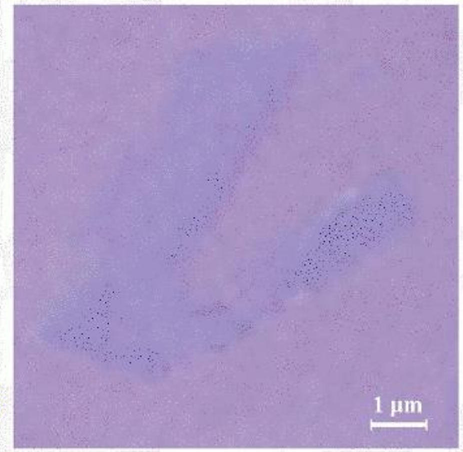
SEM



no clear signatures

Key: Visual Identification

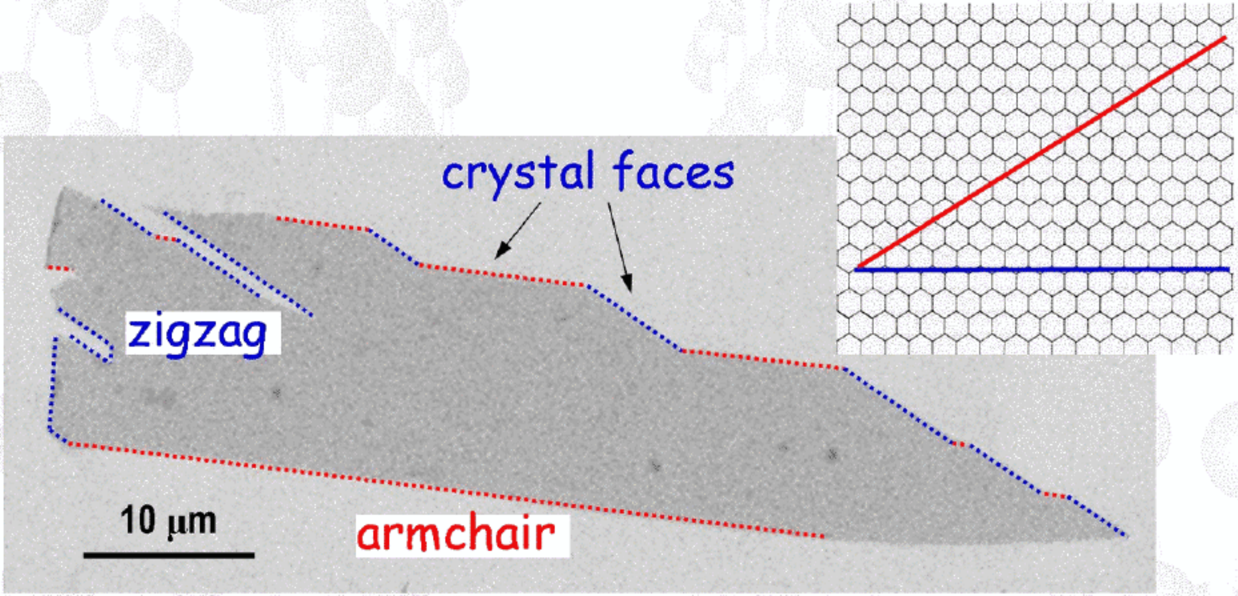
OPTICS



single layer of atoms visible by "naked" eye

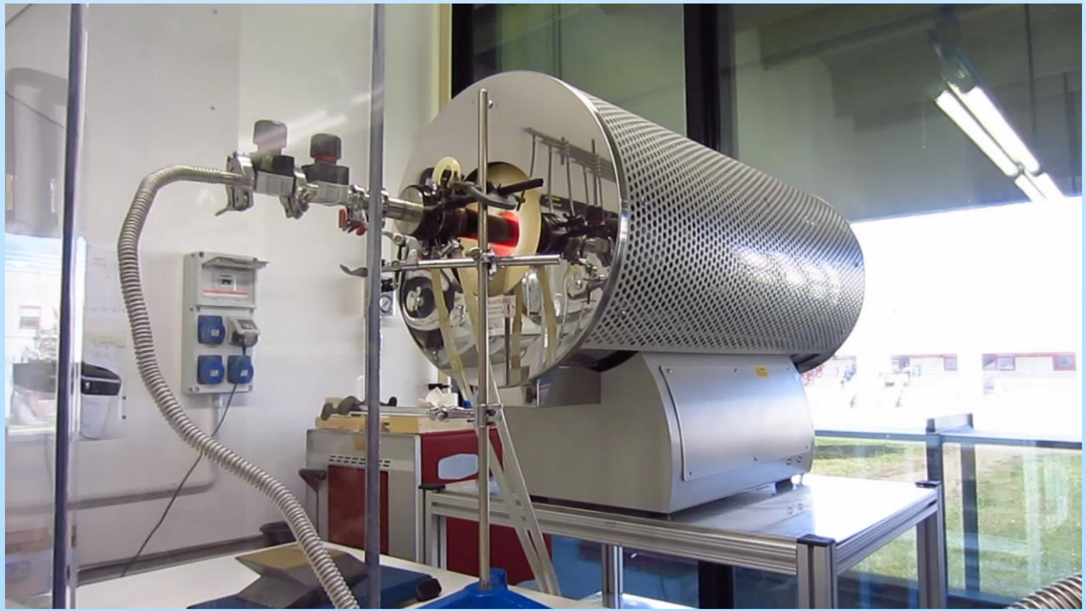
5% change in SiO_2 thickness important

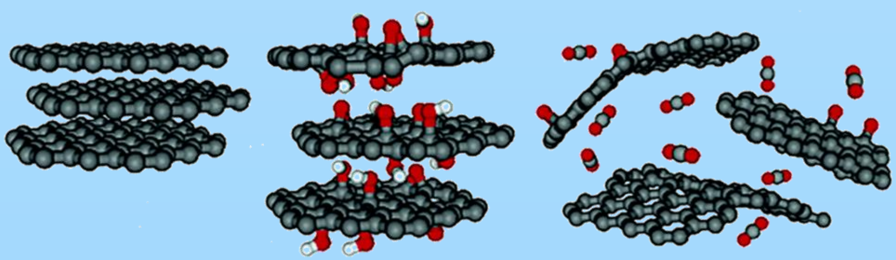
Two Dimensional Crystallites



not just flakes
but graphene crystallites

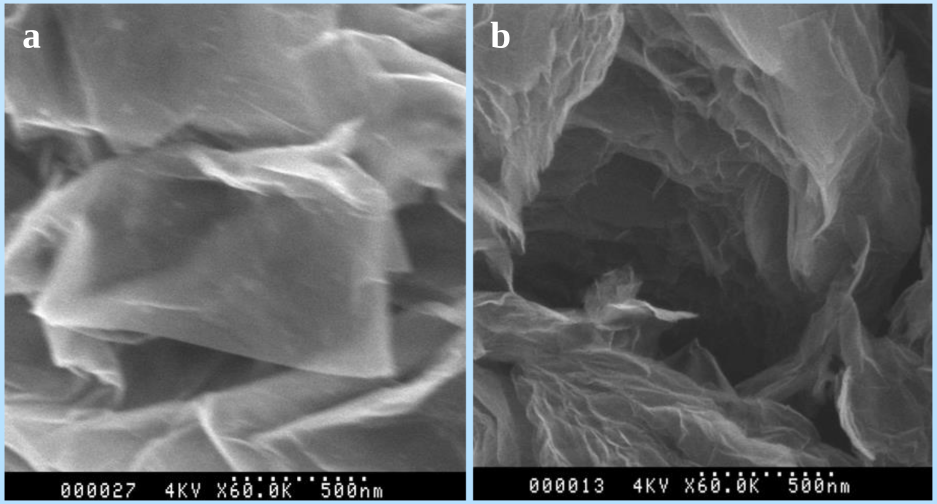
Bulk graphene (TEGO)

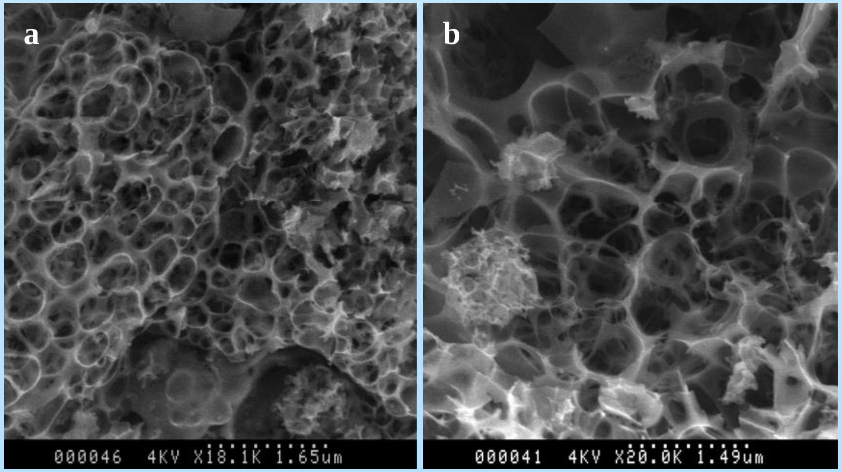


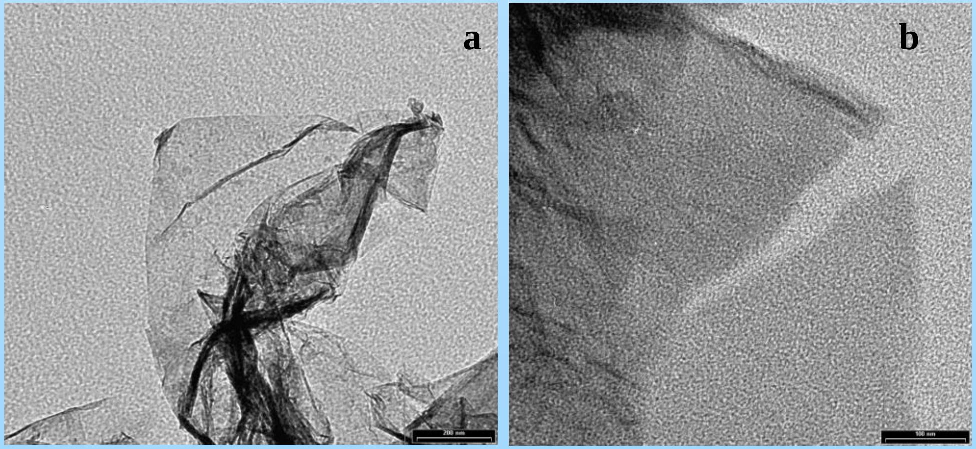


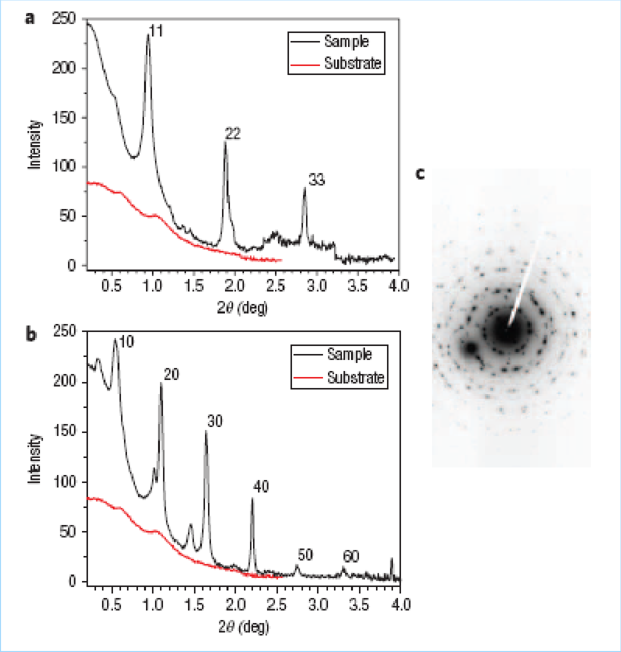
M. Riccò, D. Pontiroli et al, Nano Lett. 11, 4919 (2011)

Chemically synthesized graphene

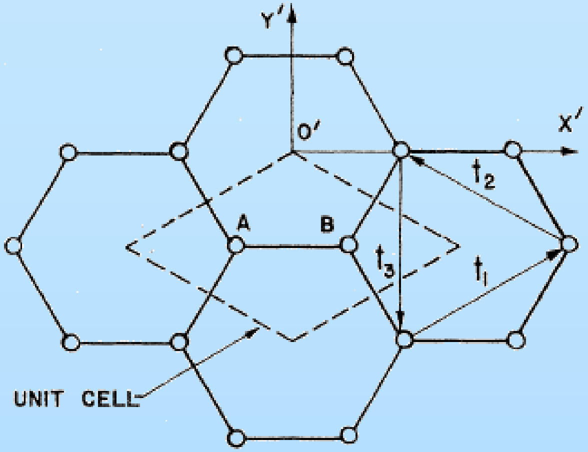


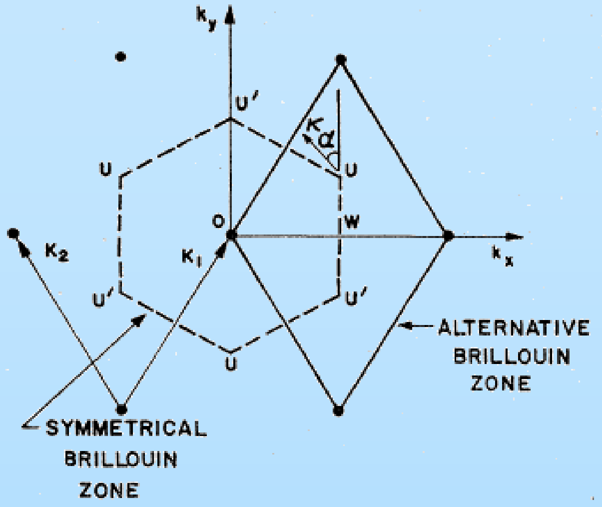




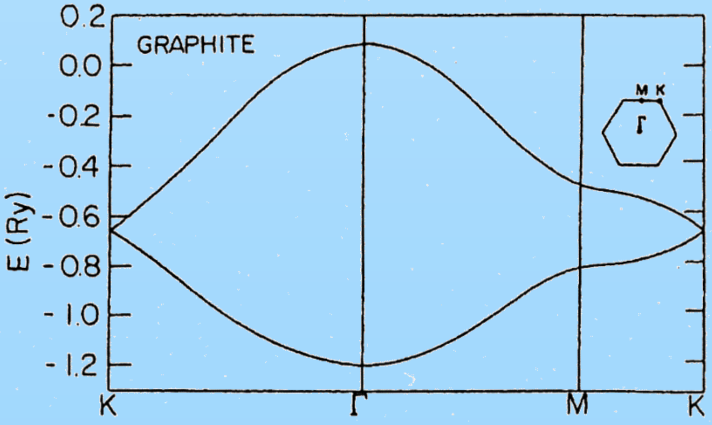


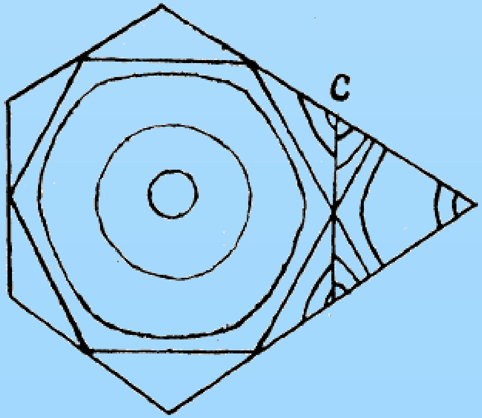
Electronic properties



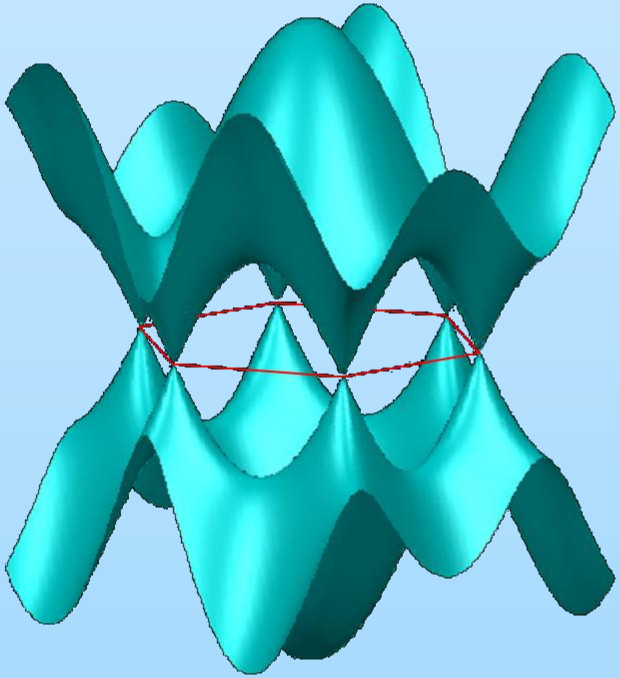


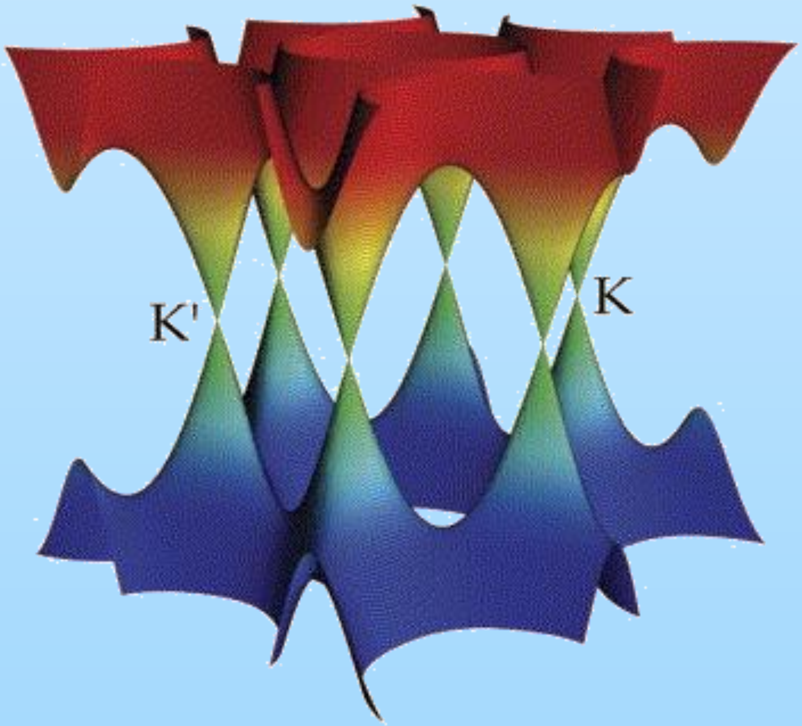
Reciprocal lattice (I Brillouin zone)

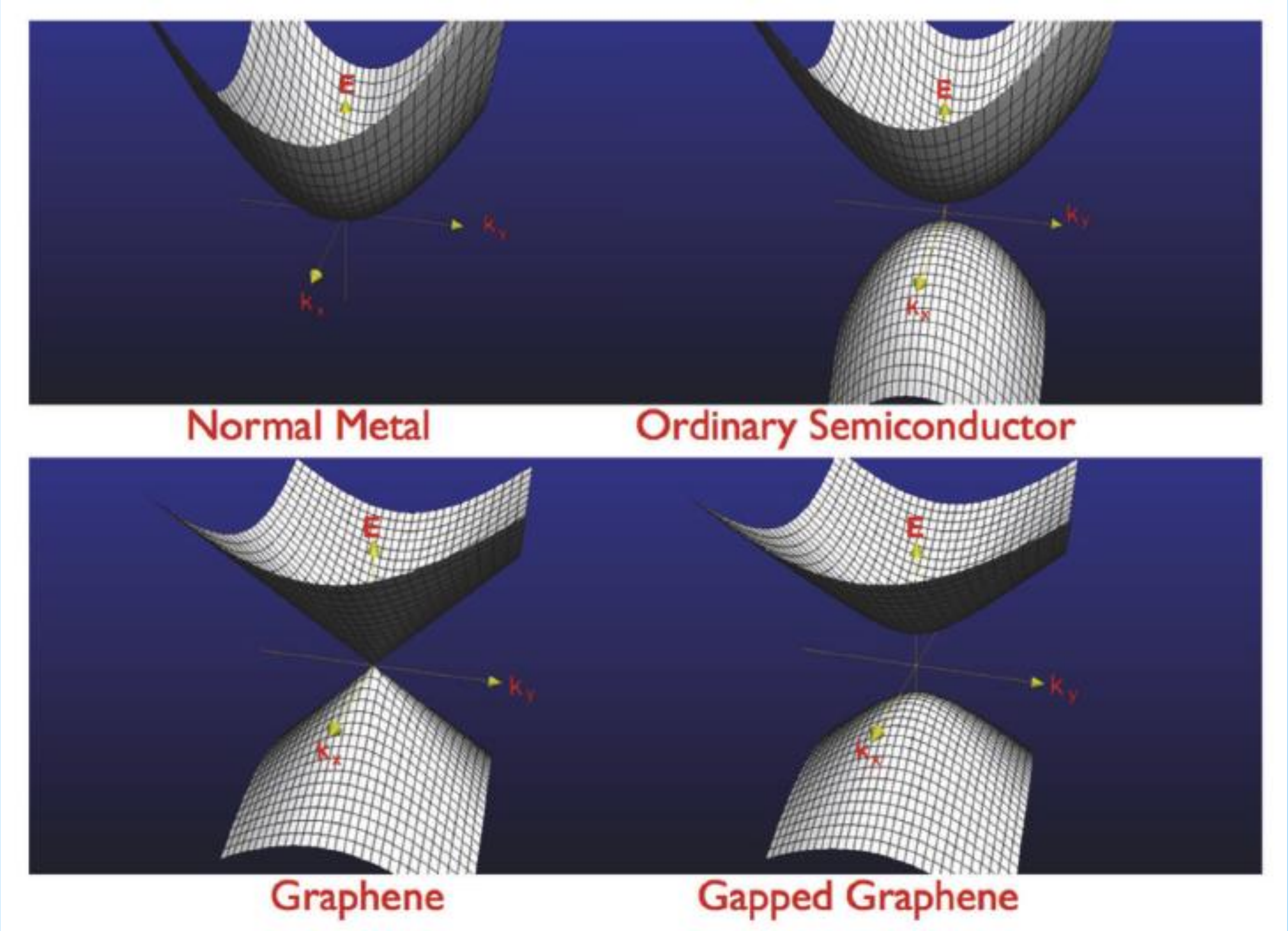




Grafene: Proprietà elettroniche






E
k
k
Normal Metal
Ordinary Semiconductor
Graphene
Gapped Graphene

Dispersion law for free particles with mass

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

Dispersion law for massless free particles

$$E = h\nu = \frac{hc}{\lambda} = \hbar c k$$

Massless electrons in graphene?

Micromechanical exfoliation of HOPG

(A.Geim & K. Novoselov, Manchester University 2004)

Vol 438|10 November 2005|**doi:10.1038/nature04233** nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry[1–3]. The ideas underlying quantum electrodynamics also influence the theory of condensed matter[4,5], but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately by the non-relativistic Schrödinger equation. Here we report an experimental study of a condensed-matter system (graphene, a single atomic layer of carbon[6,7]) in which electron transport is essentially governed by Dirac's (relativistic) equation. The charge carriers in graphene mimic relativistic particles with zero rest mass and have an effective 'speed of light' $c_* \approx 10^6\,\mathrm{m\,s^{-1}}$. Our study reveals a variety of unusual phenomena that are characteristic of two-dimensional Dirac fermions. In particular we have observed the following: first, graphene's conductivity never falls below a minimum value corresponding to the quantum unit of conductance, even when concentrations of charge carriers tend to

behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g \approx 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where n is the concentration of electrons or holes and e is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10}\,\mathrm{cm^{-2}\,V^{-1}}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10}\,\mathrm{cm^{-2}\,V^{-1}}$ for the surface charge density induced by the field effect (see the caption to Fig. 1). The agreement indicates that all the induced carriers are mobile and that there are no trapped charges in graphene. From the linear dependence $\sigma(V_g)$ we found carrier mobilities $\mu = \sigma/ne$, which reached $15{,}000\,\mathrm{cm^2\,V^{-1}\,s^{-1}}$ for both electrons and holes, were independent of temperature T between 10 and 100 K and were probably still limited by defects in parent graphite.

To characterize graphene further, we studied Shubnikov-de Haas oscillations (SdHOs). Figure 2 shows examples of these oscillations for different magnetic fields B, gate voltages and temperatures. Unlike ultrathin graphite[7], graphene exhibits only one set of SdHO for both

Electrons in graphene behave like particles with zero rest mass and a velocity

$c^* = v_F = 10^6$ m/s :

1- Even when the concentration of charge carriers tends to zero, the conductivity never falls below a minimum value corresponding to one quantum unit of conductance.

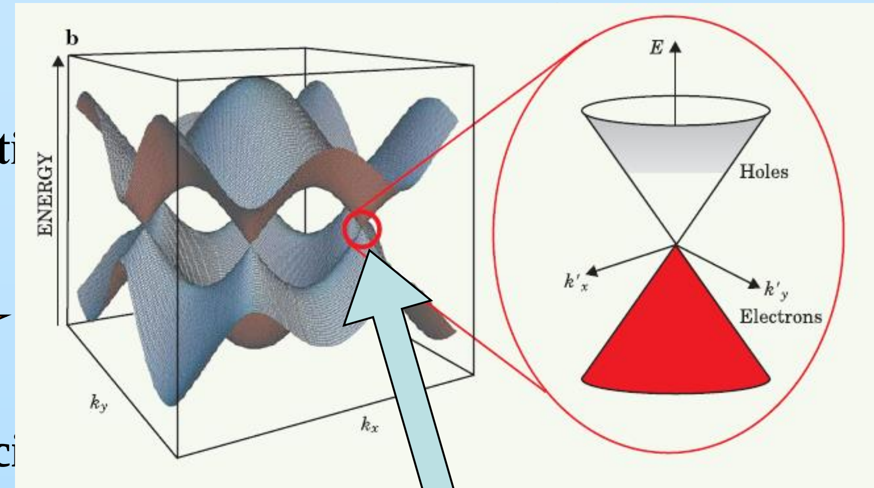
2- The quantum Hall effect is anomalous: it exhibits half-integer filling factors.

3- The cyclotron mass m_c of the carriers is related to their energy by the relation $E = m_c c^{*2}$

Fermioni di Dirac

Relativistic quantum mechanics → Dirac equati

$$v_F \vec{\sigma} \cdot \vec{\nabla} \psi(\mathbf{r}) = E\psi(\mathbf{r})$$



1- Particles of spin ½ have antiparticles associ with them.

2- Charge conjugation symmetry (same spinor for e^- and e^+)

3- The energy spectrum of particles with mass has a gap $2E_0 = 2mc^2$

4- When $E >> E_0$ $E=c(h/2\pi)\,k$

5- When m=0 E~k for whatever energy. The elicity (chirality) is defined.

Conical energy spectrum $E=(h/2\pi)\,k\,v_F$

Charge conjugation (electrons ↔ holes)

$\vec{\sigma}$ → Sublattice A↔B

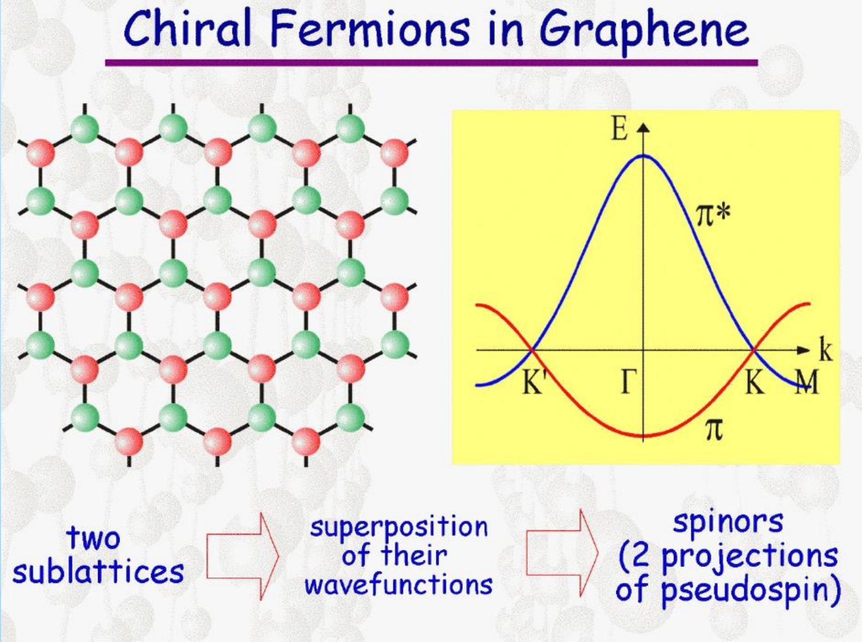
(pseudospin)

Dirac Fermions

In matter, electrons and holes generally behave differently.

In graphene they show perfect symmetry following the charge conjugation symmetry rule

The spinor wavefunction in QED is replaced by the pseudospin σ which identifies the sublattice (A or B) → chirality



Grafene: Trasporto

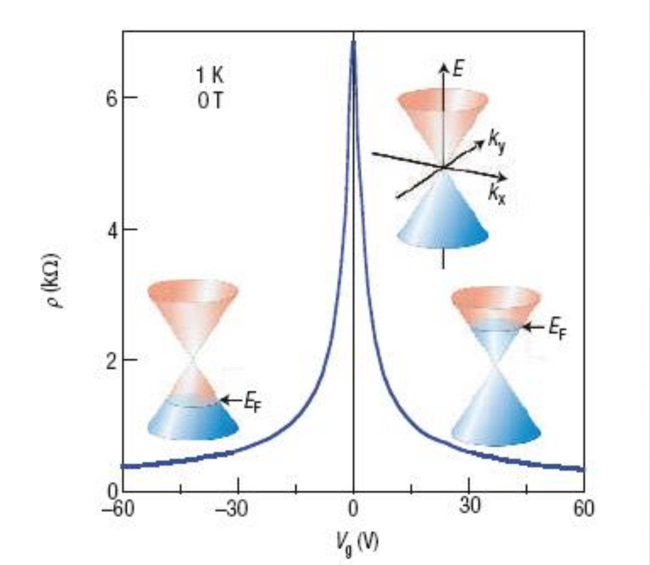
0<n<10^{13}cm^{-2} sia e che h

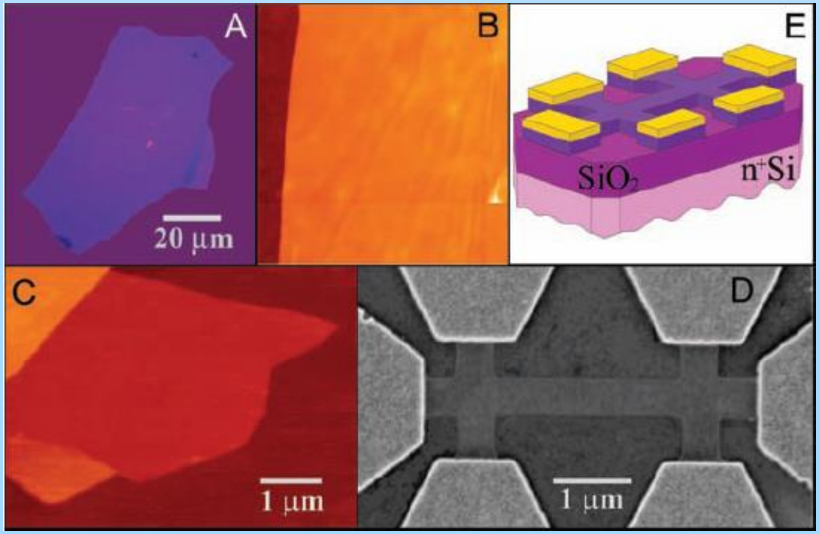
Mobility: $\mu = \sigma / n\, e$ ~15000 cm^{2}V^{-1}s^{-1}

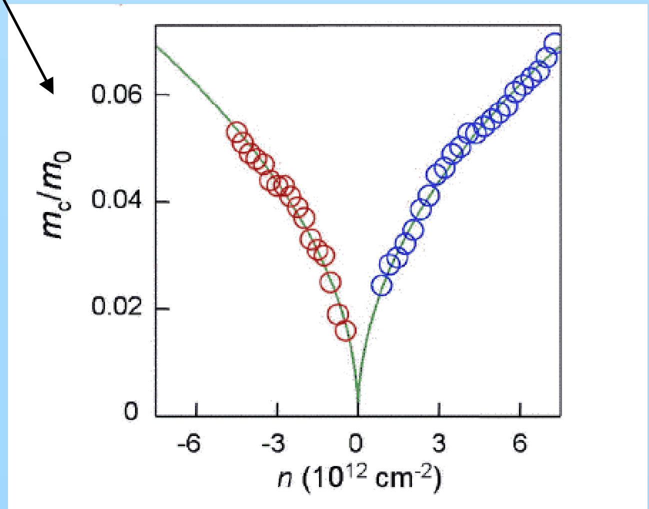
(indip. from T e da n, determined by impurities↔semiconductors)

Cyclotron mass m_c

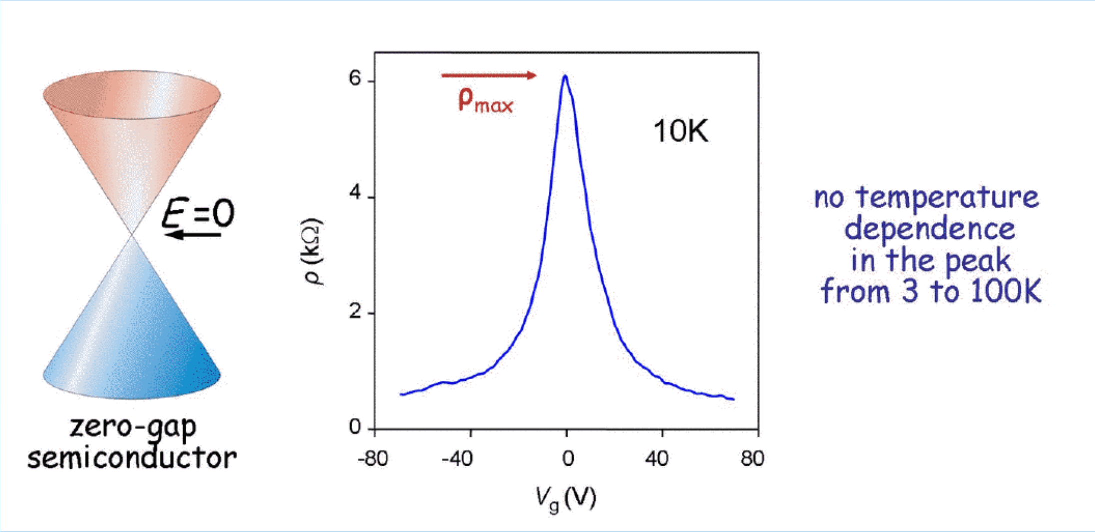
$v_F = 10^6$ m/s

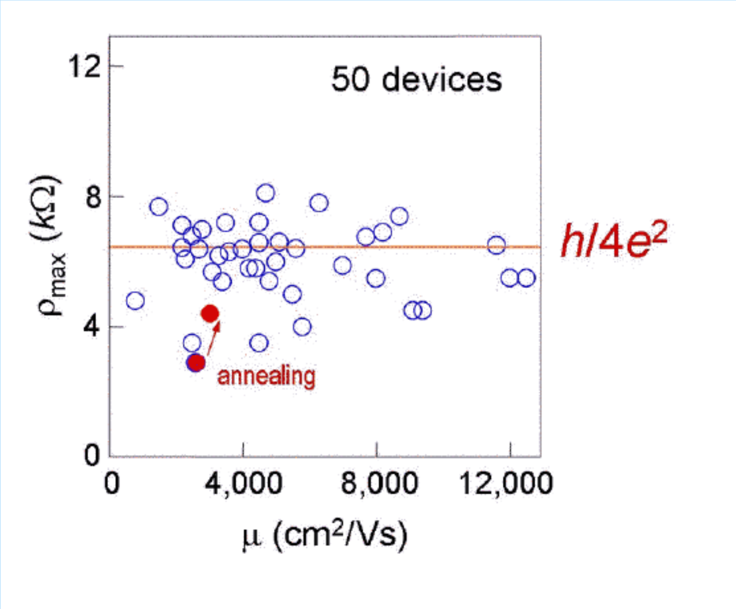


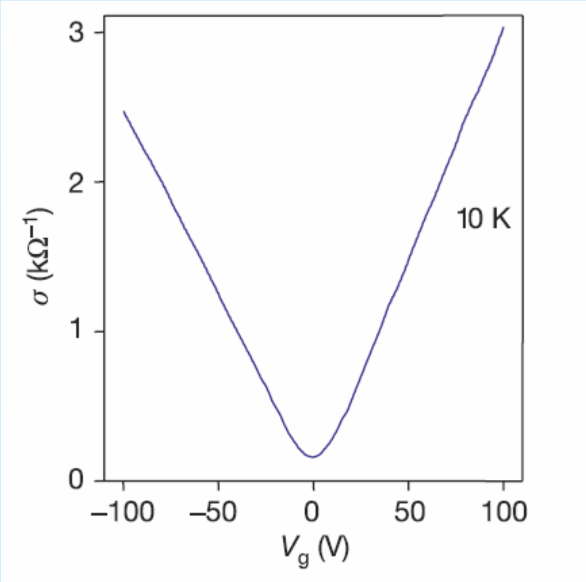




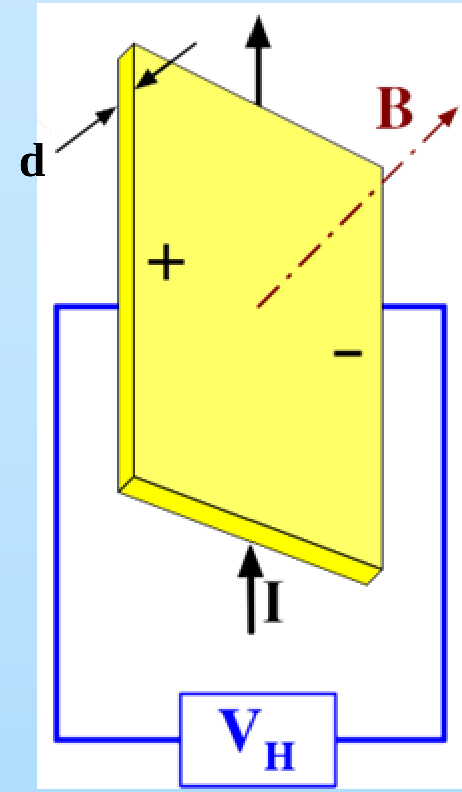
Graphene: Transport







Hall effect



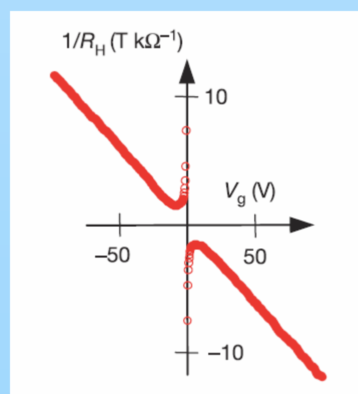
$$V_H = \frac{-IB/d}{ne}.$$

$$R_H = \frac{E_y}{j_x B} = \frac{V_H}{IB/d} = -\frac{1}{ne},$$

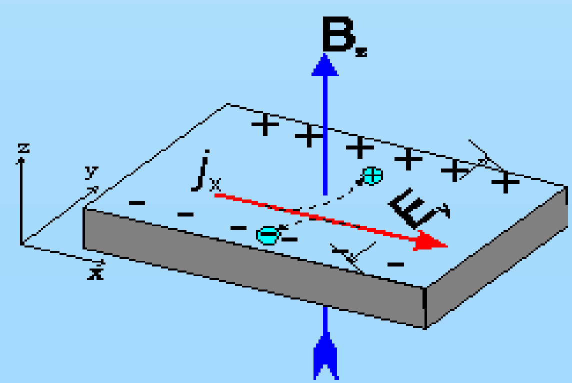
$$R_H = \frac{1}{(p-n)e}$$

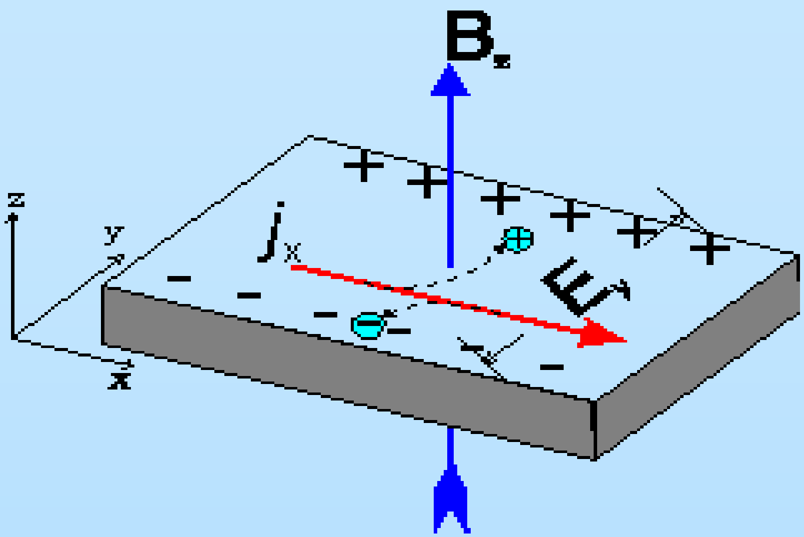
Semicondctors

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{B}{ne}$$



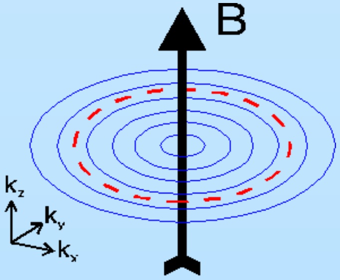
Graphene



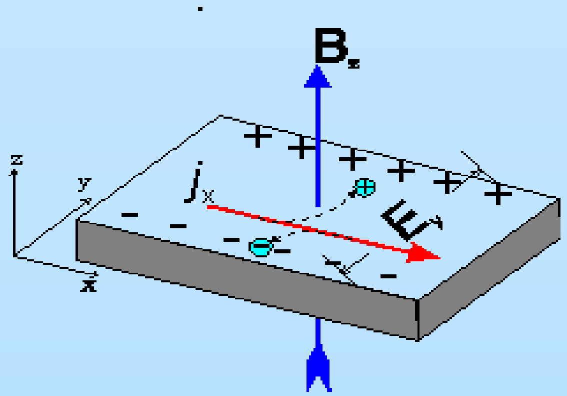

B_z
z
y
x
j_x
E

Quantum Hall Effect

Nobel Prize 1985 Klaus von Klitzig



$$\omega_c = \frac{eB}{m^*}$$

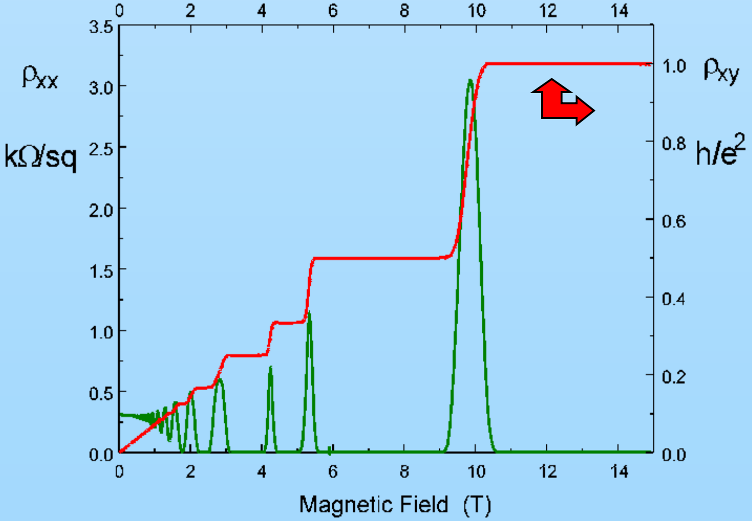


$$E = E_i + (N + \frac{1}{2})\hbar\omega_c$$

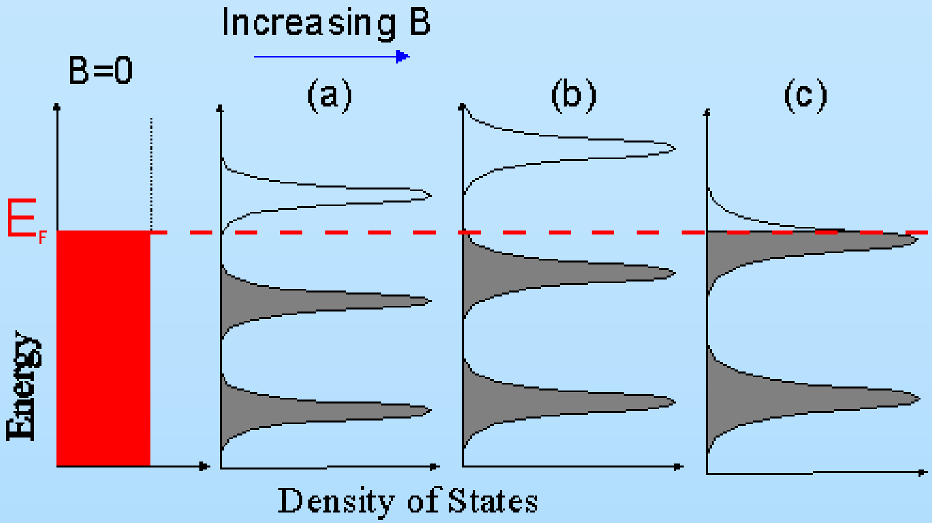
$$n = \frac{eB}{h}$$

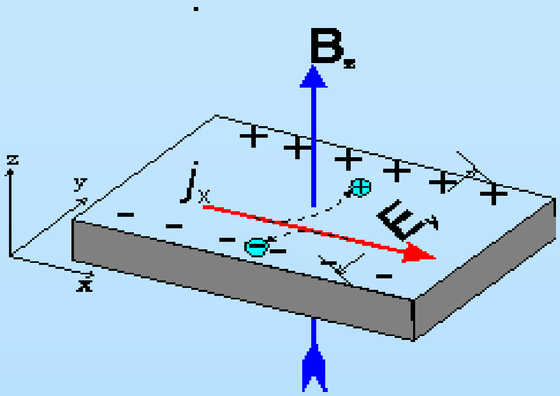
Nr. of states for each Landau level

$$\rho_{xy} = \frac{B}{ne} = \frac{h}{ieB}\frac{B}{e} = \frac{1}{i}\frac{h}{e^2}$$



Quantum Hall Effect

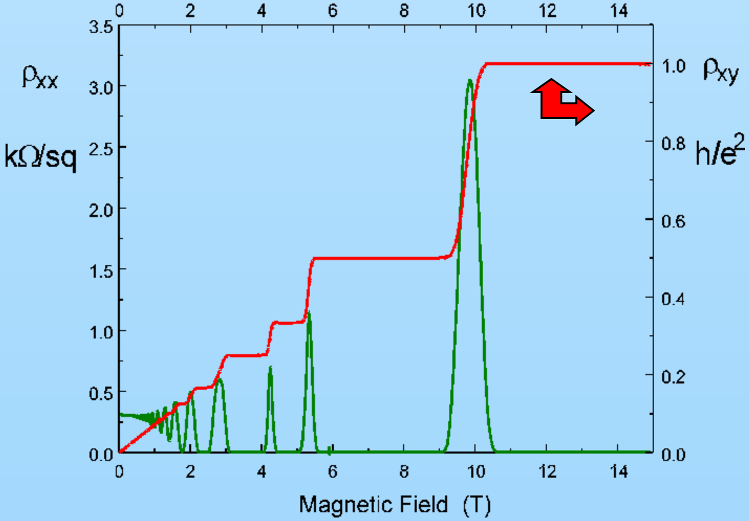




$$n = \frac{eB}{h}$$

Nr. of states for each Landau level

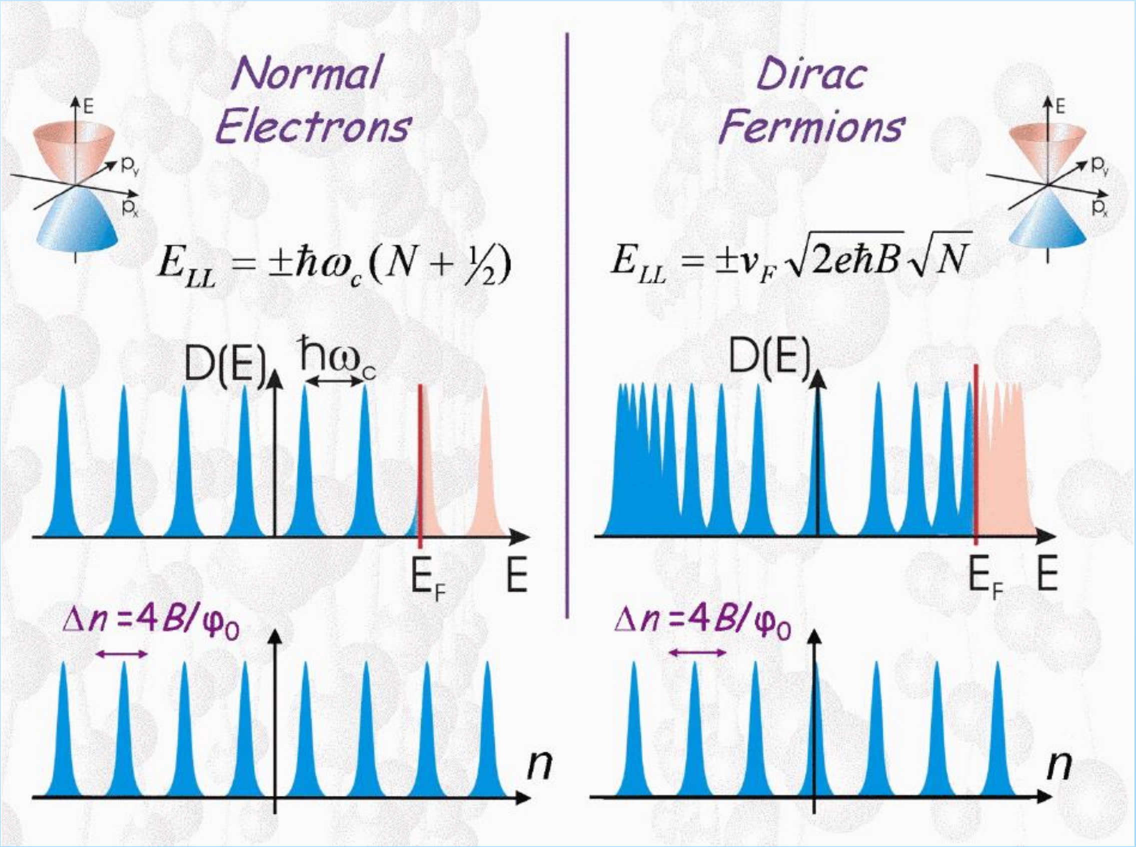
$$\rho_{xy} = \frac{B}{ne} = \frac{h}{ieB}\frac{B}{e} = \frac{1}{i}\frac{h}{e^2}$$



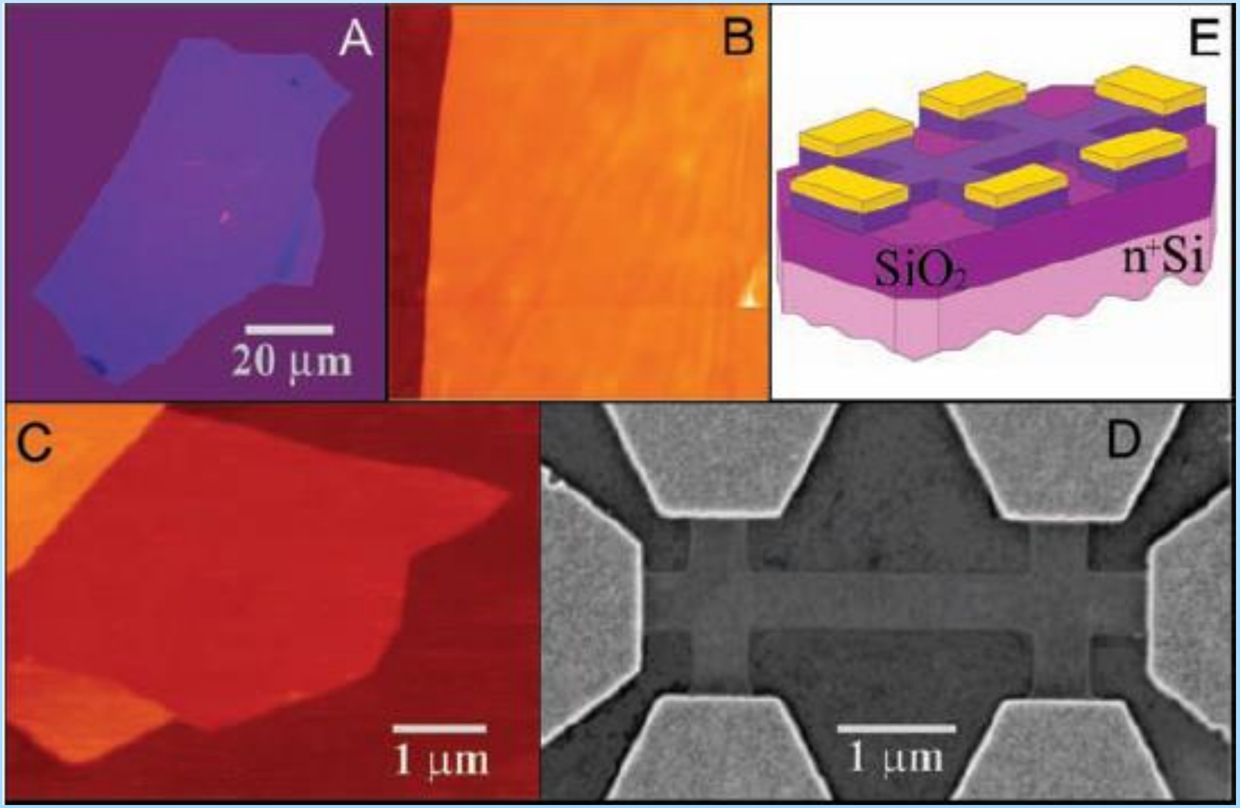
eterojunction GaAs-GaAlAs at 30mK

Quantum Hall Effect with Dirac Electron

Dirac Fermions m=0 → $$E_{\nu\sigma} = \pm\sqrt{2e\hbar B v_F^2 N}$$

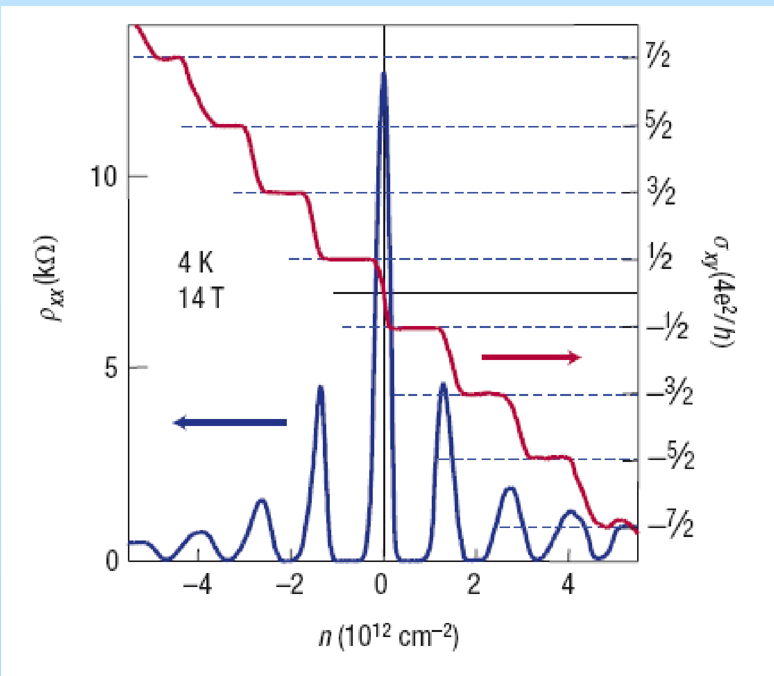


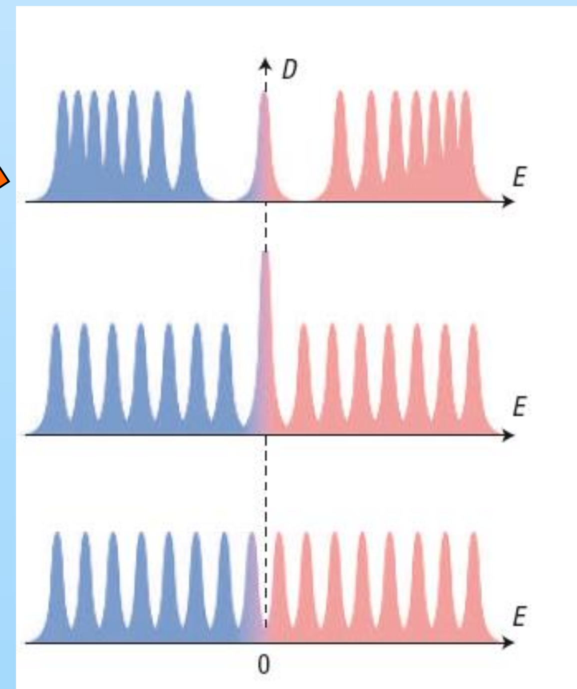
Quantum Hall Effect with Dirac Electron



Anomalous Quantum Hall Effect

Dirac Fermions → $$E_{\nu\sigma} = \pm\sqrt{2e\hbar B v_F^2 N}$$ **(QED)**





Monolayer

Bilayer A-A

(chiral particle with mass: oximoron)

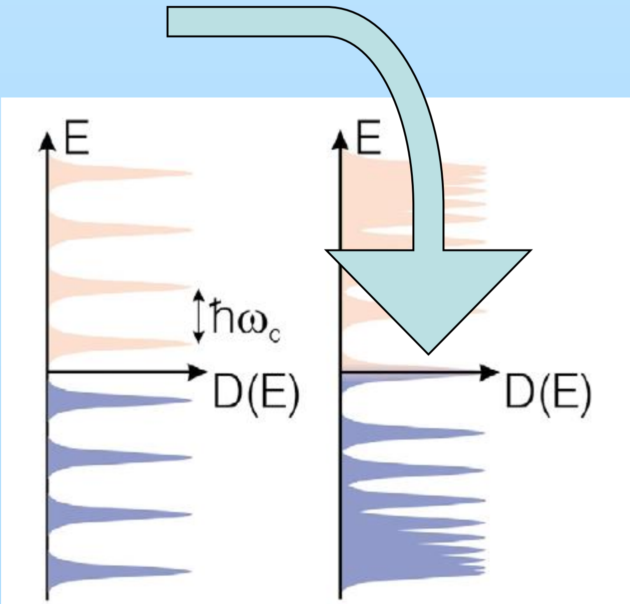
Bilayer A-B

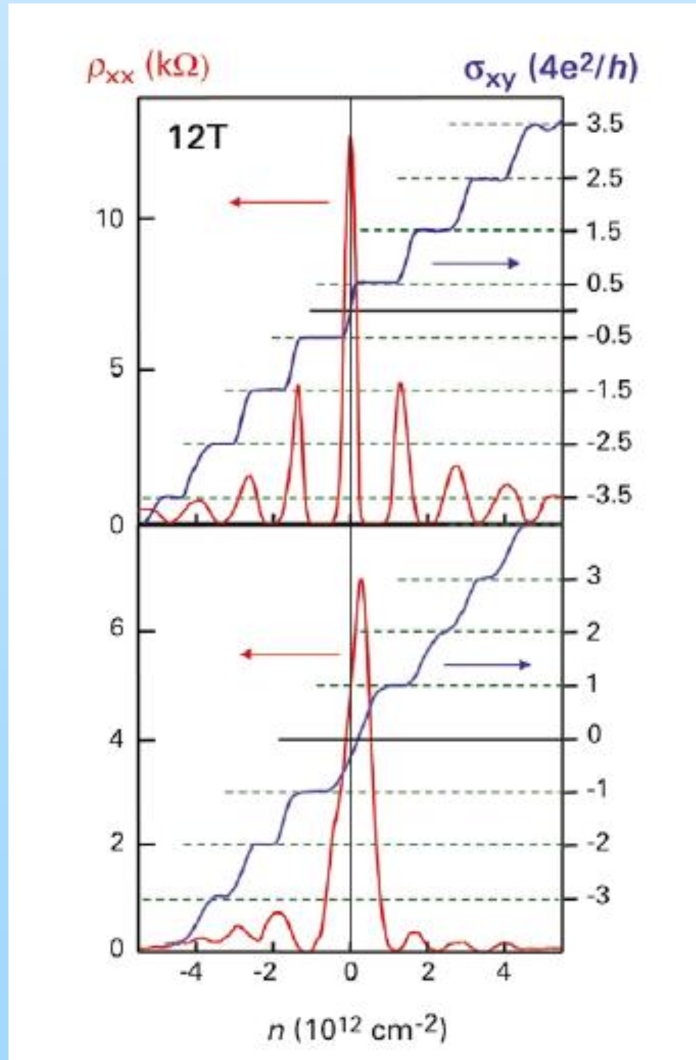
(gap)

K. S. Novoselov et al. Nature 438 (2005) 197

Singolo layer vs. bilayer

The origin of fractional quantum plateaus is in the existence of a state at E=0





Singolo

Bilayer A-B

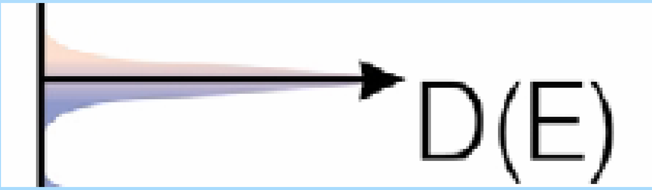
$$E_\nu \sim \sqrt{\nu(\nu - 1)}$$

Effect of ripples?

Atiyah-Singer theorem (T. of superstrings)→

Being the states at E=0 chiral, they are stable for gauge fields and space curvatures.

Ripples (B inhomogeneities up to 1T) do NOT inhibit anomalous QHE

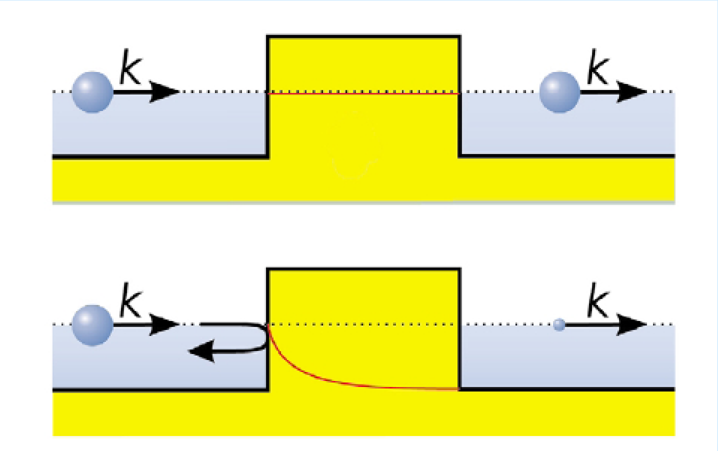


Quantum Tunneling

In semiconductors the tunneling probability decreases exponentially with the height and width of the barrier (when $\Delta E<e-h$).

Resonant tunnel: when E(e)=E(h) (in the barrier)

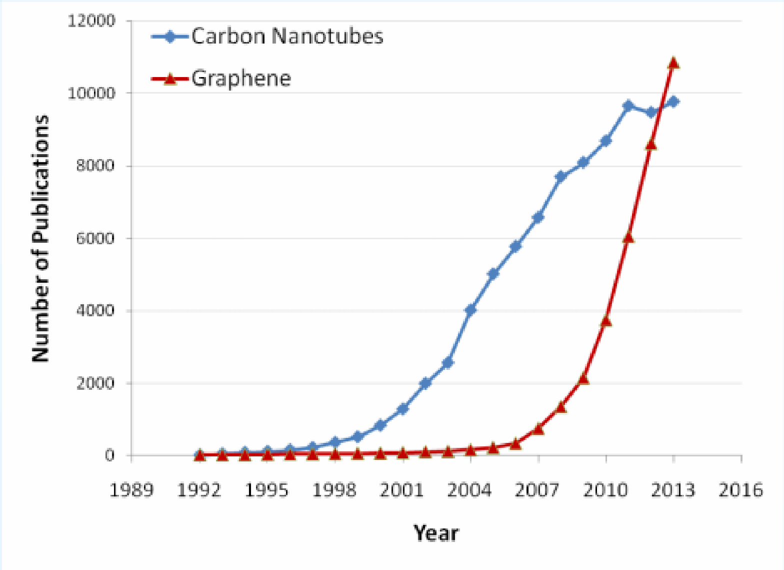
In graphene T=1



Klein paradox (QED)

A barrier $2m_ec^2$ high allows the transmission of the el. through the formation of an e-p pair.

Reformulation of the Heisenberg principle in QED


Carbon Nanotubes
Graphene
Number of Publications
12000
10000
8000
6000
4000
2000
0
1989
1992
1995
1998
2001
2004
2007
2010
2013
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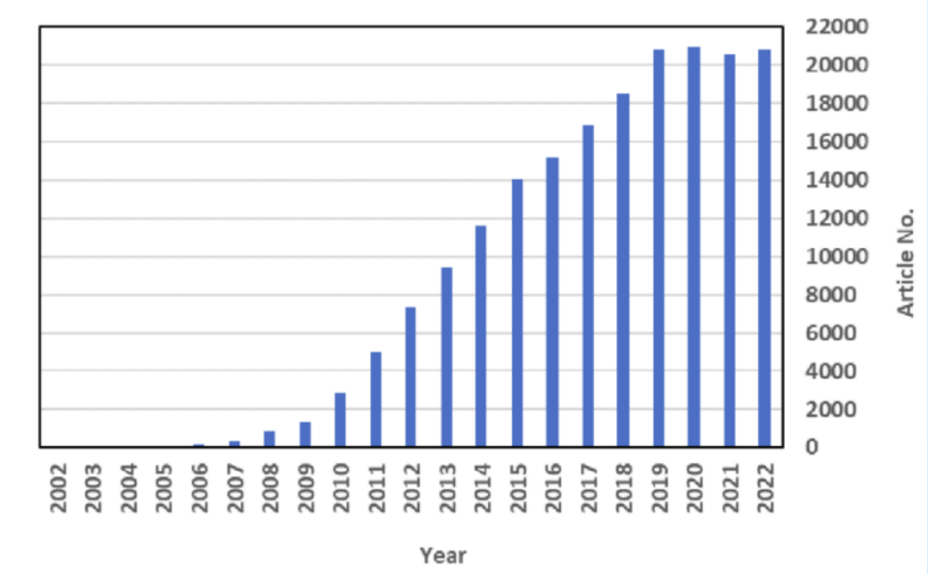
Year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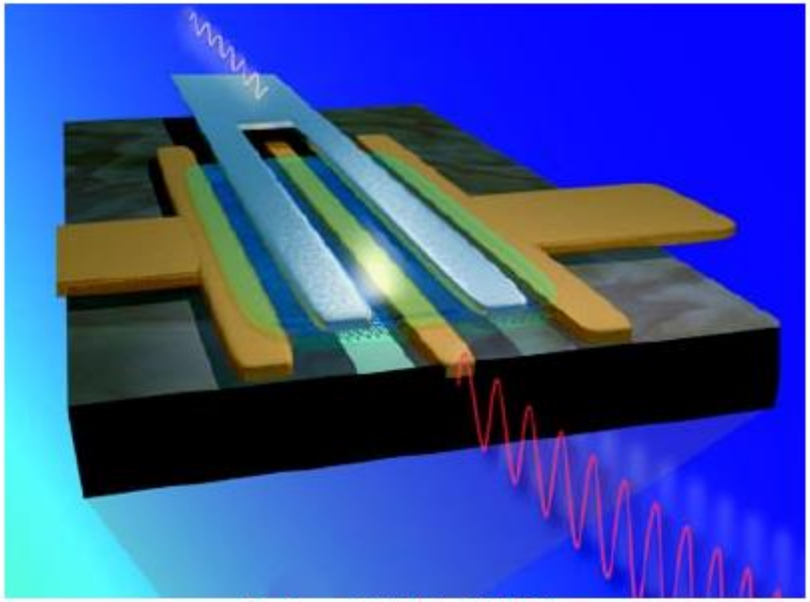

Graphene FET hits 100 GHz

Physicists in the US have made the fastest graphene transistor ever, with a cut-off frequency of 100 GHz. The device can be further miniaturized and optimized so that it could soon outperform conventional devices made from silicon, says the team. The transistor could find application in microwave communications and imaging systems.

Graphene – a sheet of carbon just one atom thick – shows great promise for use in electronic devices because electrons can move through it at extremely high speeds. This is because they behave like relativistic particles with no rest mass. This, and other unusual physical and mechanical properties, means that the "wonder material" could replace silicon as the electronic material of choice and might be used to make faster transistors than any that exist today.

CAPTION: IBM Scientists Demonstrate World's Fastest Graphene Transistor. (PRNewsFoto/IBM)
LOCATION: YORKTOWN HEIGHTS, NY, UNITED STATES
POST DATE: Feb/5/2010 7:59 AM
TAG ID: prnphotos089381
FORMAT: 7.3" x 9.0" @ 300 DPI (2219 x 2700 Color JPEG)
SPECIAL: SEE STORY 20100205/NY50316, NY Media contact: Michael Loughran, IBM, +1-914-945-1613, mloughra@us.ibm.com.
Document: IBM GRAPHENE TRANSISTOR
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