



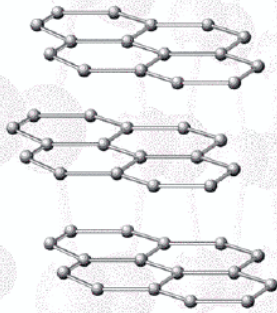
Grafene e fermioni di Dirac

M. Riccò



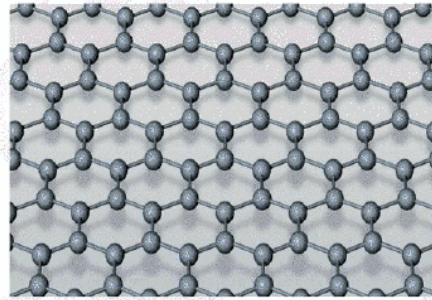
GRAPHENE ALLOTROPES

3D



Graphite

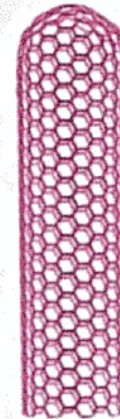
2D



graphene

**PRESUMED
NOT TO EXIST
IN THE FREE STATE**

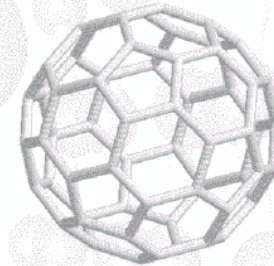
1D



**Carbon
Nanotube**

multi-wall:
1952 to *Iijima* 1991
single-wall: 1993

0D



Buckyballs

Kroto et al 1985



PHYSICAL REVIEW

VOLUME 176, NUMBER 1

5 DECEMBER 1968

Crystalline Order in Two Dimensions*

N. D. Mermin[†]

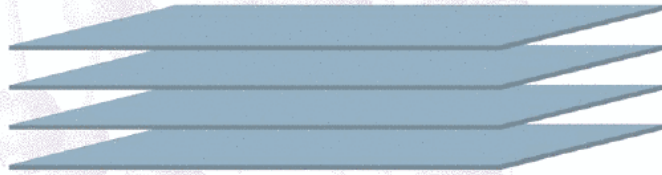
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

(Received 1 July 1968)

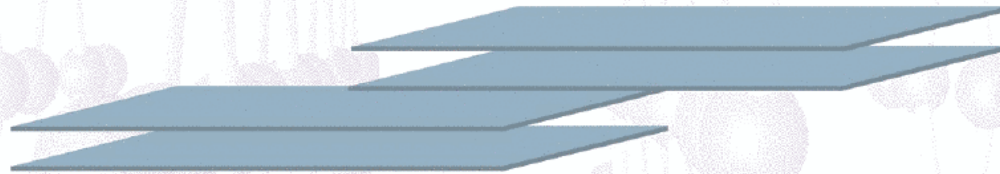
If N classical particles in two dimensions interacting through a pair potential $\Phi(\vec{r})$ are in equilibrium in a parallelogram box, it is proved that every $\vec{k} \neq 0$ Fourier component of the density must vanish in the thermodynamic limit, provided that $\Phi(\vec{r}) - \lambda r^2 |\nabla^2 \Phi(\vec{r})|$ is integrable at $r = \infty$ and positive and nonintegrable at $r = 0$, both for $\lambda = 0$ and for some positive λ . This result excludes conventional crystalline long-range order in two dimensions for power-law potentials of the Lennard-Jones type, but is inconclusive for hard-core potentials. The corresponding analysis for the quantum case is outlined. Similar results hold in one dimension.



Extracting a Single Plane



GRAPHITE IS
STRONGLY LAYERED



SLICE DOWN TO
ONE ATOMIC PLANE



individual atomic sheets: do they exist?



Produzione del grafene

Scissione micromeccanica della HOPG

(A.Geim & K. Novoselov, Manchester University 2004)

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,¹ A. K. Geim,^{1*} S. V. Morozov,² D. Jiang,¹
Y. Zhang,¹ S. V. Dubonos,² I. V. Grigorieva,¹ A. A. Firsov²

We describe monocrystalline graphitic films, which are a few atoms thick but are nonetheless stable under ambient conditions, metallic, and of remarkably high quality. The films are found to be a two-dimensional semimetal with a tiny overlap between valence and conductance bands, and they exhibit a strong ambipolar electric field effect such that electrons and holes in concentrations up to 10^{13} per square centimeter and with room-temperature mobilities of $\sim 10,000$ square centimeters per volt-second can be induced by applying gate voltage.

The ability to control electronic properties of a material by externally applied voltage is at the heart of modern electronics. In many cases, it is the electric field effect that allows one to vary the carrier concentration in a semiconductor device and, consequently, change an electric current through it. As the

semiconductor industry is nearing the limits of performance improvements for the current technologies dominated by silicon, there is a constant search for new, nontraditional materials whose properties can be controlled by the electric field. The most notable recent examples of such materials are organic conductors (1) and carbon nanotubes (2). It has long been tempting to extend the use of the field effect to metals [e.g., to develop all-metallic transistors that could be scaled down to much smaller sizes and would consume less energy and operate at higher frequencies

¹Department of Physics, University of Manchester, Manchester M13 9PL, UK. ²Institute for Microelectronics Technology, 142432 Chernogolovka, Russia.

*To whom correspondence should be addressed.
E-mail: geim@man.ac.uk



Stabilità meccanica

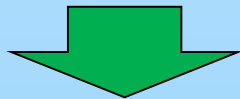
Teorema di Mermin-Wagner → Nessun ordine a lungo range in 2D

Dinamica → in 2D il nr. di fononi a grossa lungh. d'onda diverge

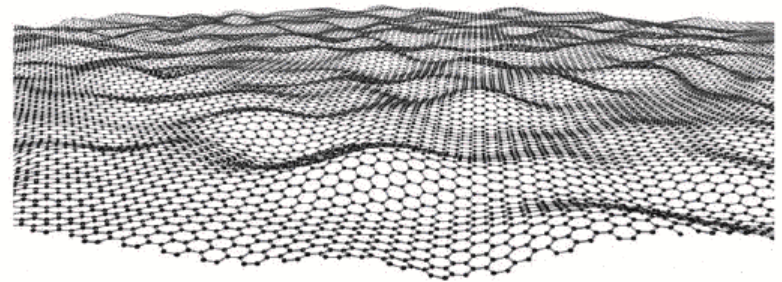
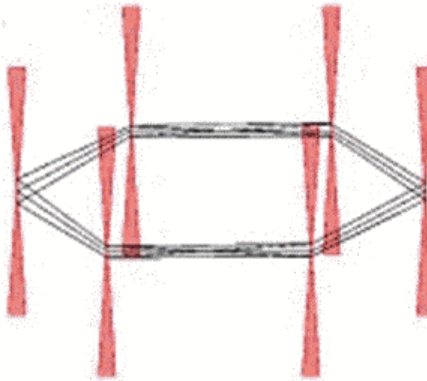
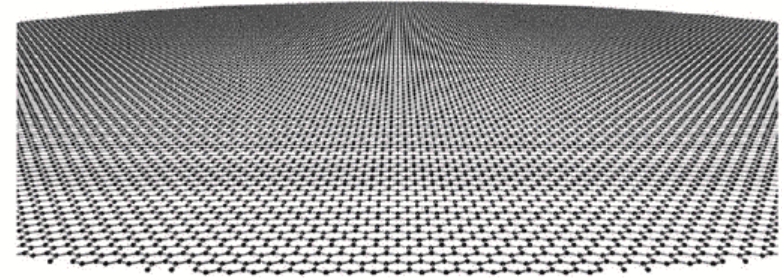
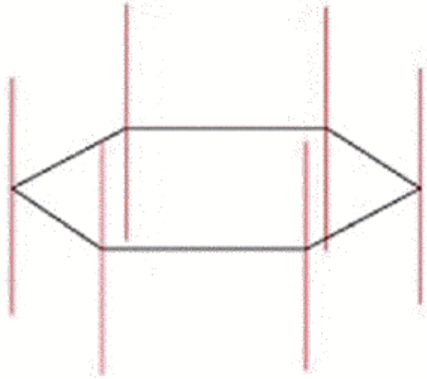
Criterio di Lindemann $\langle u \rangle \sim 0.1d$ fusione



Termini anarmonici di accoppiamento tra modi di bending e stretchnig sopprimono queste fluttuazioni distruttive.



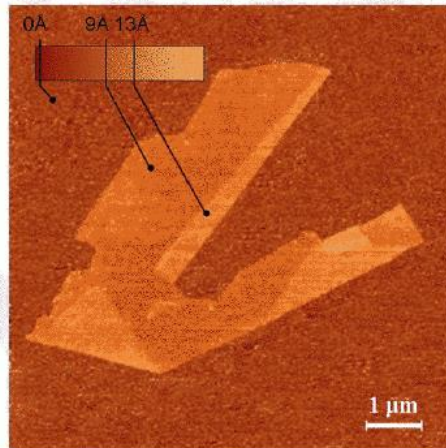
Il grafene presenta “ripples”



cones in Fourier space rather than rods

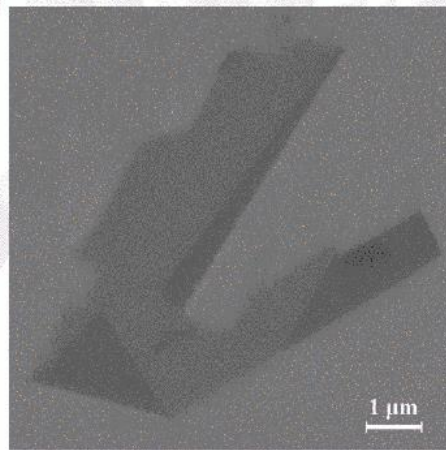


Free-Standing Graphene



AFM

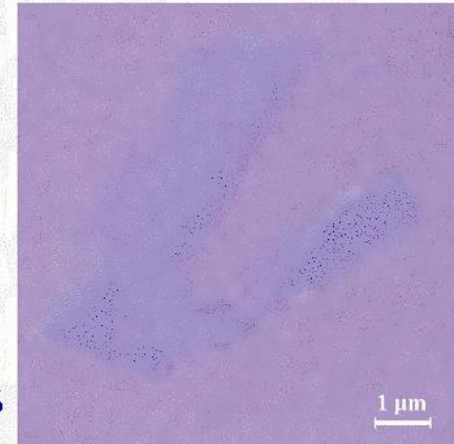
low throughput



SEM

no clear signatures

Key: Visual Identification



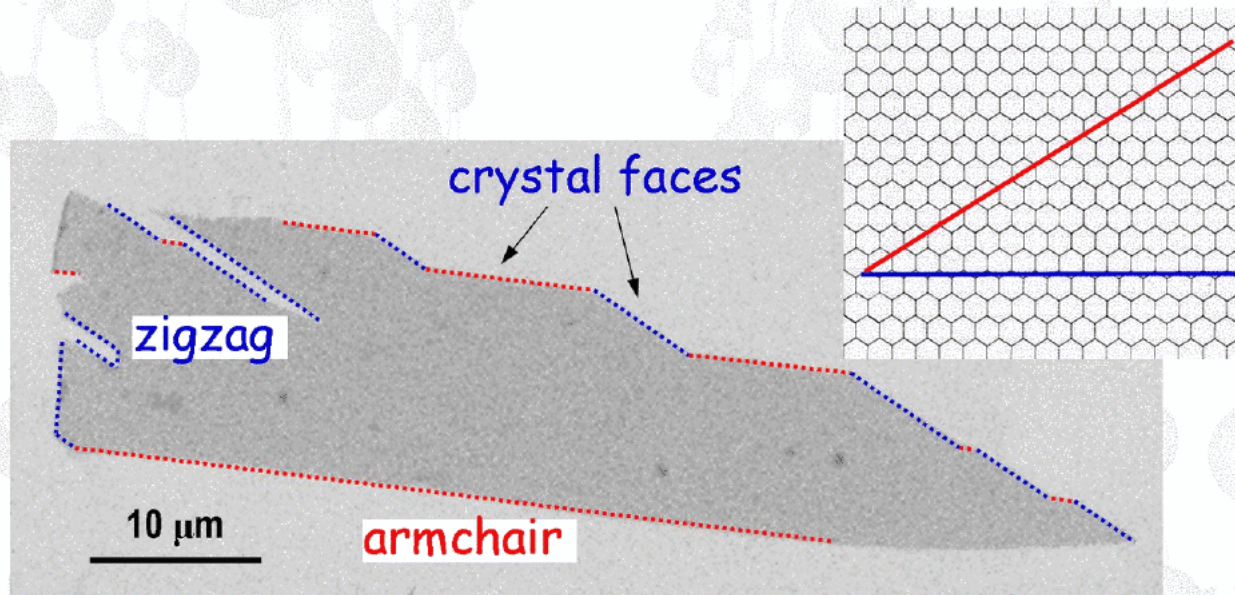
OPTICS

single layer of atoms
visible by "naked" eye

5% change in SiO_2 thickness
important

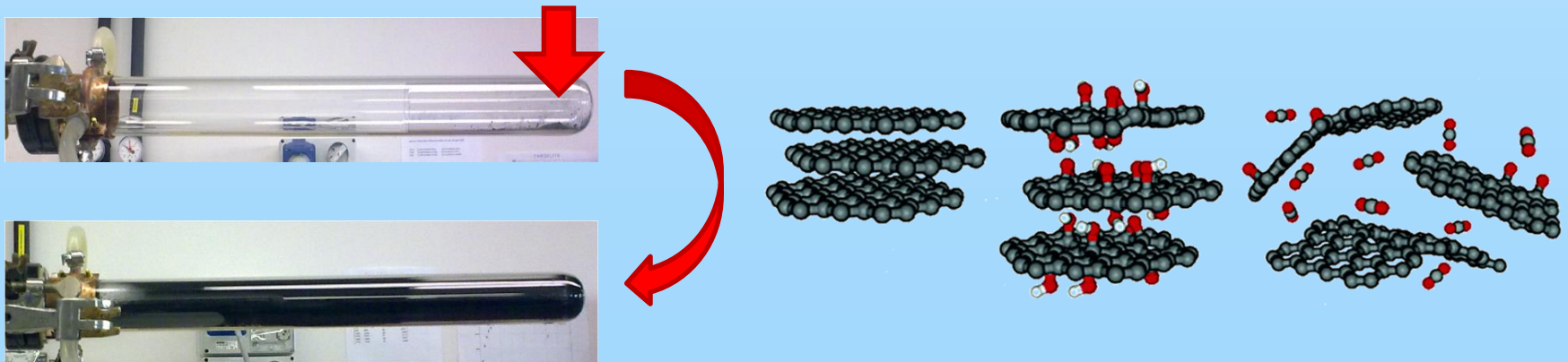
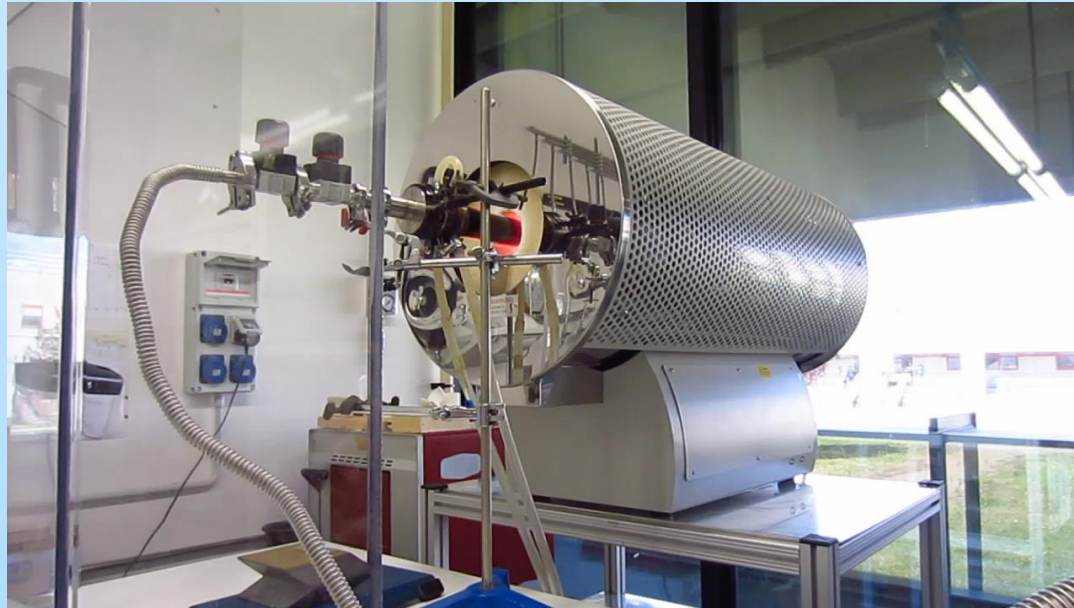


Two Dimensional Crystallites



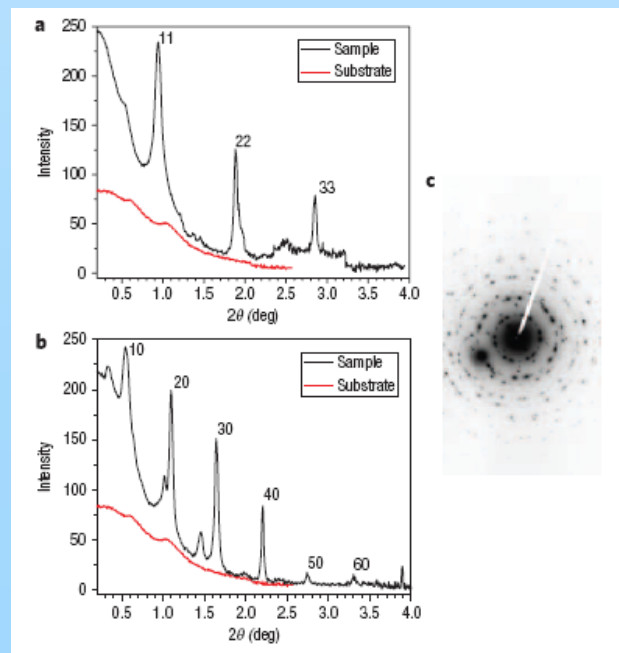
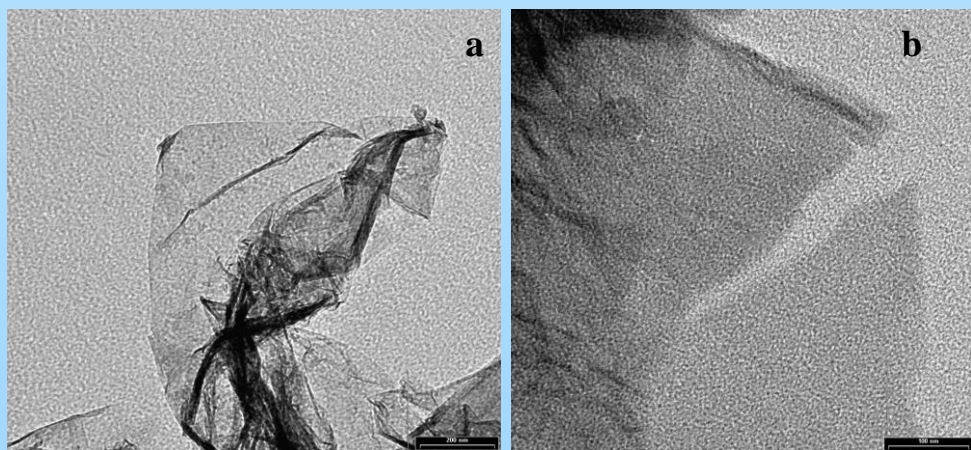
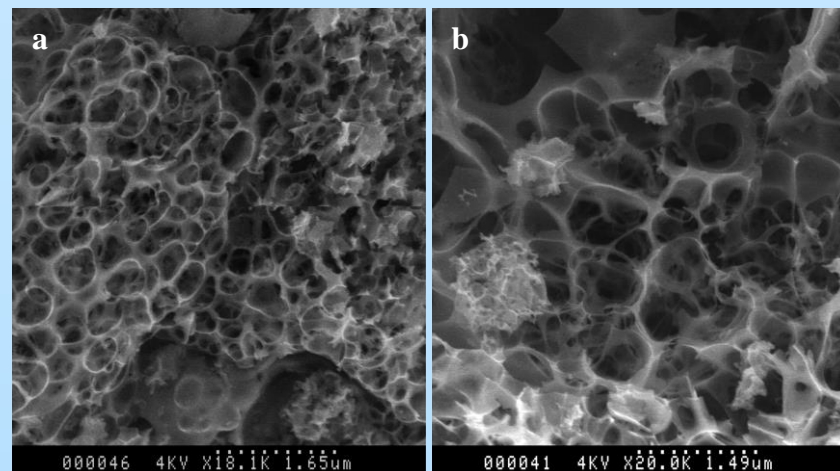
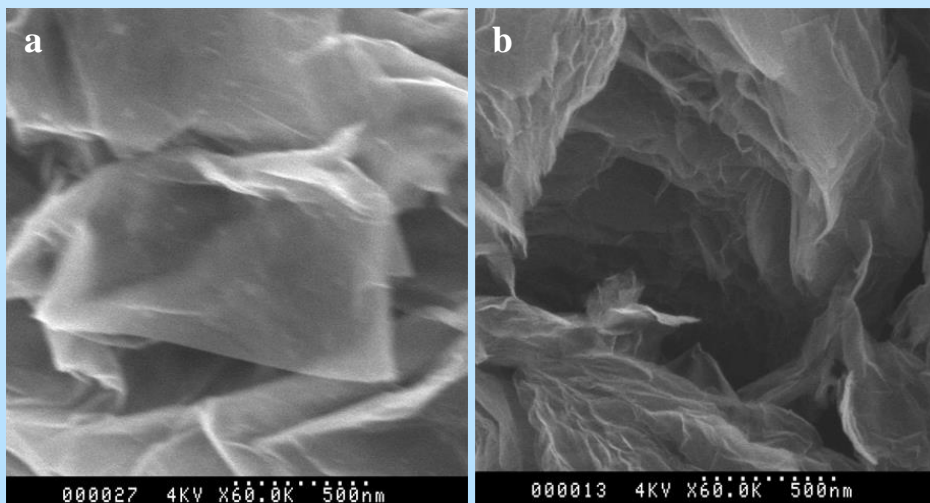
not just flakes
but graphene crystallites

Bulk graphene (TEGO)



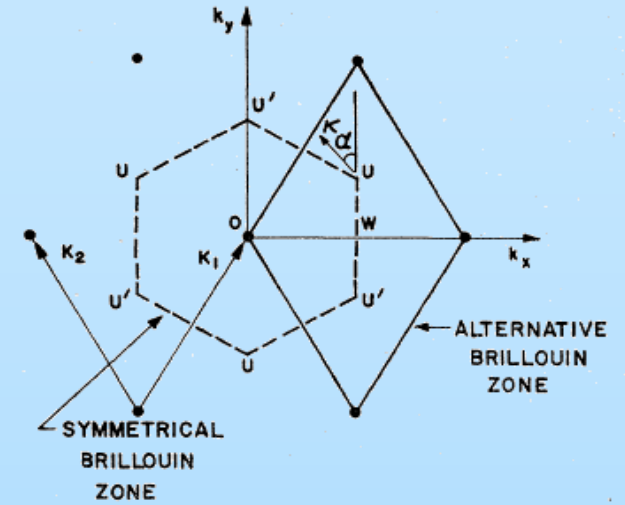
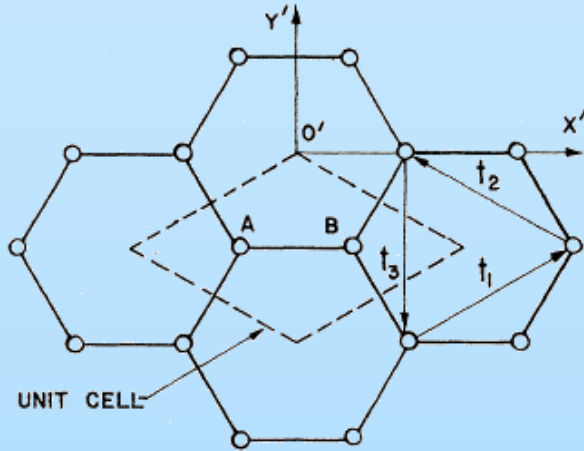


Grafene sintetizzato chimicamente

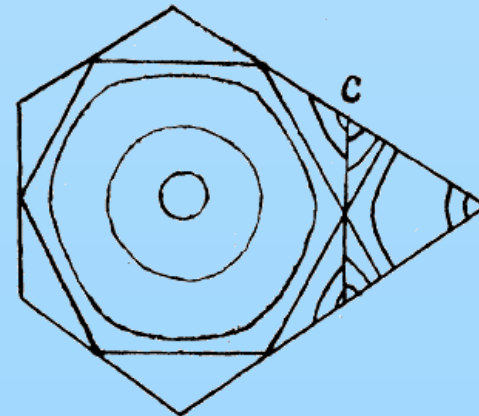
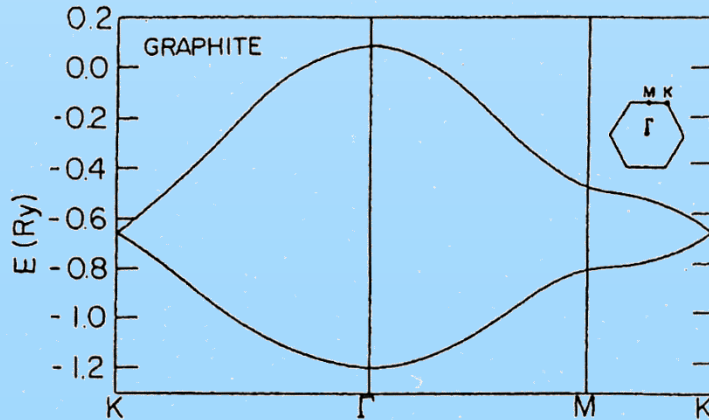




Proprietà elettroniche

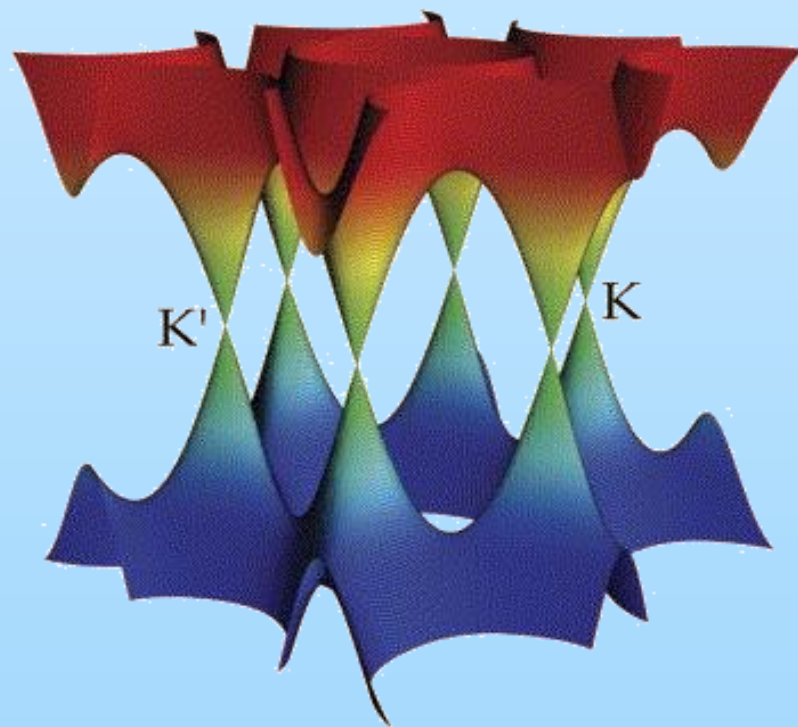
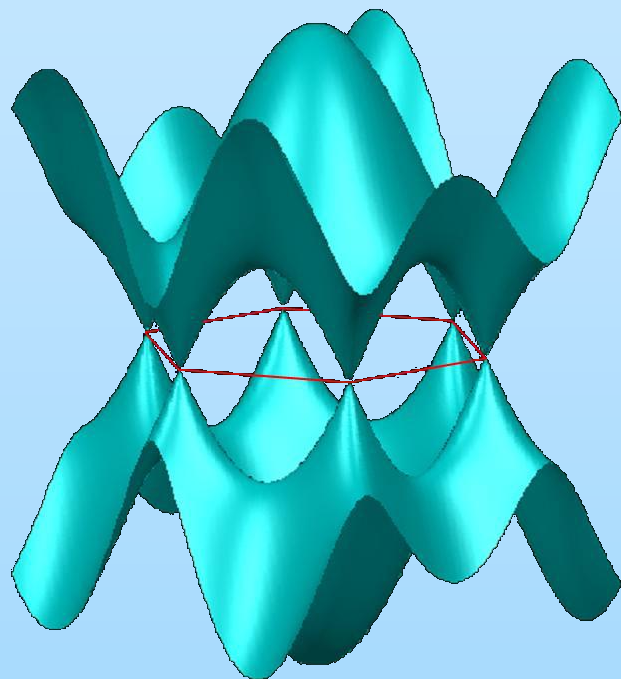


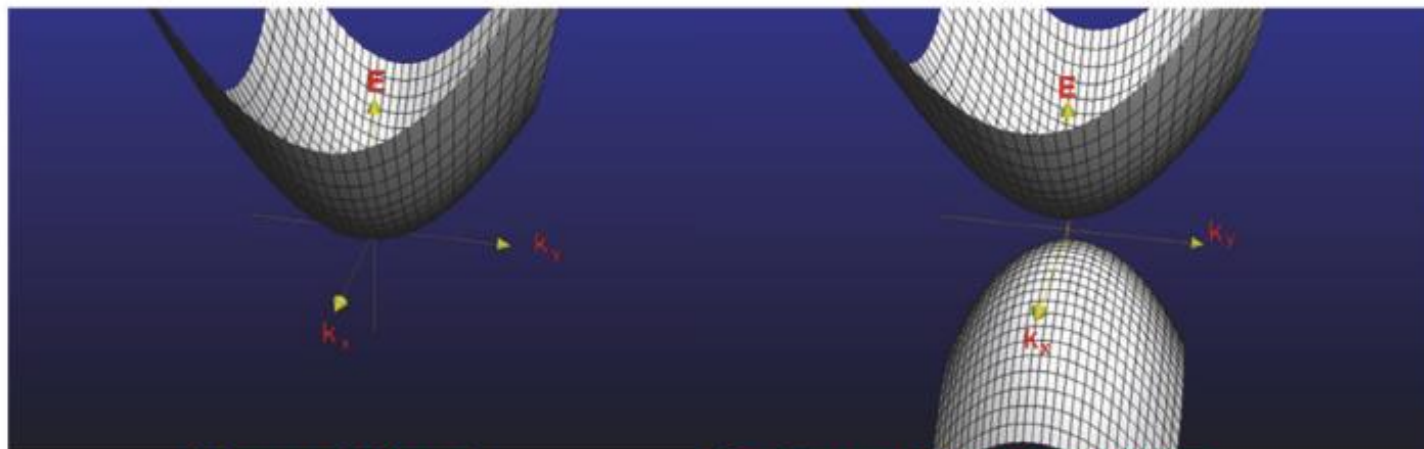
Reticolo reciproco (I zona di B.)





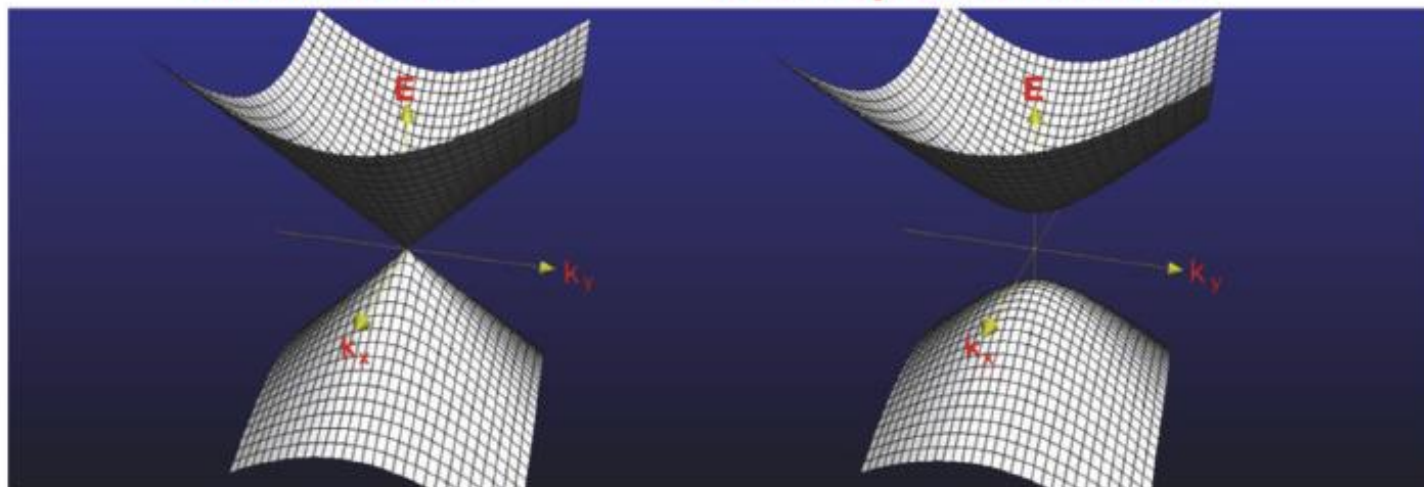
Grafene: Proprietà elettroniche





Normal Metal

Ordinary Semiconductor

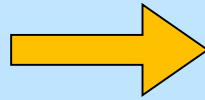


Graphene

Gapped Graphene

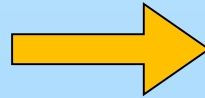


**Dispersione per
particelle libere
dotate di massa**



$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

**Dispersione per
particelle libere **non**
dotate di massa**



$$E = h\nu = \frac{hc}{\lambda} = \hbar ck$$



Elettroni a massa=0 nel grafene?

Scissione micromeccanica della HOPG

(A.Geim & K. Novoselov, Manchester University 2004)

Vol 438|10 November 2005|doi:10.1038/nature04233

nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov¹, A. K. Geim¹, S. V. Morozov², D. Jiang¹, M. I. Katsnelson³, I. V. Grigorieva¹, S. V. Dubonos² & A. A. Firsov²

Quantum electrodynamics (resulting from the merger of quantum mechanics and relativity theory) has provided a clear understanding of phenomena ranging from particle physics to cosmology and from astrophysics to quantum chemistry¹⁻³. The ideas underlying quantum electrodynamics also influence the theory of condensed matter^{4,5}, but quantum relativistic effects are usually minute in the known experimental systems that can be described accurately by the non-relativistic Schrödinger equation. Here we report an experimental study of a condensed-matter system (graphene, a single atomic layer of carbon^{6,7}) in which electron transport is essentially governed by Dirac's (relativistic) equation. The charge carriers in graphene mimic relativistic particles with zero rest mass and have an effective 'speed of light' $c_* \approx 10^6 \text{ m s}^{-1}$. Our study reveals a variety of unusual phenomena that are characteristic of two-dimensional Dirac fermions. In particular we have observed the following: first, graphene's conductivity never falls below a minimum value corresponding to the quantum unit of conductance, even when concentrations of charge carriers tend to

behaviour shows that substantial concentrations of electrons (holes) are induced by positive (negative) gate voltages. Away from the transition region $V_g \approx 0$, Hall coefficient $R_H = 1/ne$ varies as $1/V_g$, where n is the concentration of electrons or holes and e is the electron charge. The linear dependence $1/R_H \propto V_g$ yields $n = \alpha V_g$ with $\alpha \approx 7.3 \times 10^{10} \text{ cm}^{-2} \text{ V}^{-1}$, in agreement with the theoretical estimate $n/V_g \approx 7.2 \times 10^{10} \text{ cm}^{-2} \text{ V}^{-1}$ for the surface charge density induced by the field effect (see the caption to Fig. 1). The agreement indicates that all the induced carriers are mobile and that there are no trapped charges in graphene. From the linear dependence $\sigma(V_g)$ we found carrier mobilities $\mu = \sigma/ne$, which reached $15,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ for both electrons and holes, were independent of temperature T between 10 and 100 K and were probably still limited by defects in parent graphite.

To characterize graphene further, we studied Shubnikov-de Haas oscillations (SdHOs). Figure 2 shows examples of these oscillations for different magnetic fields B , gate voltages and temperatures. Unlike ultrathin graphite⁷, graphene exhibits only one set of SdHO for both



Gli elettroni nel grafene si comportano come particelle di massa a riposo nulla dotate di una velocità $c^* = v_F = 10^6$ m/s :

1- Anche quando la concentrazione dei portatori di carica tende a zero, la conducibilità non scende mai al di sotto di un valore minimo corrispondente ad una unità quantistica di conduttanza.

2- L'effetto Hall quantistico è anomalo: mostra fattori di riempimento semi interi.

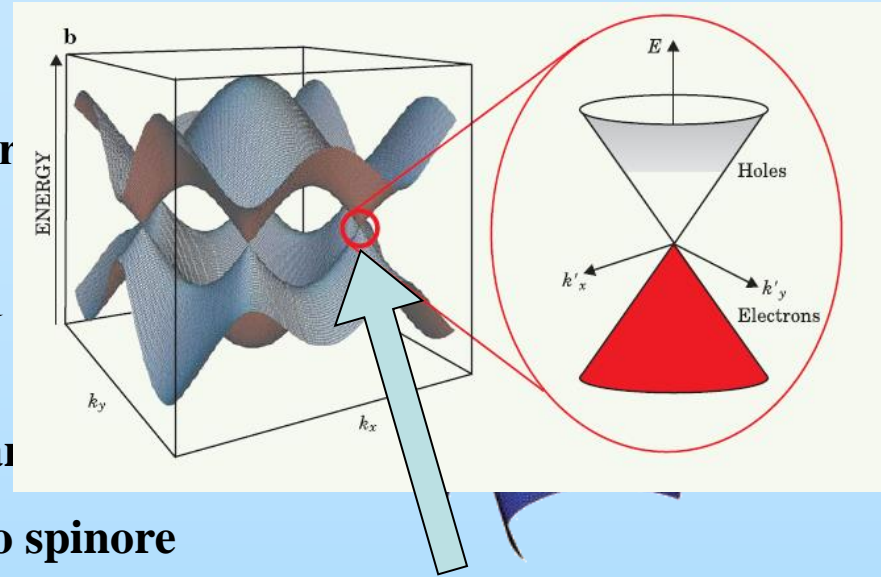
3- La massa ciclotronica m_c dei portatori è legata alla loro energia dalla relazione $E = m_c c^{*2}$



Fermioni di Dirac

Meccanica quantistica relativistica → Eq. di Dirac

$$v_F \vec{\sigma} \cdot \vec{\nabla} \psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad \Downarrow$$



1- A particelle di spin $1/2$ sono associate antiparticle

2- Simmetria di coniugazione di carica (stesso spinore per e^- e e^+)

3- Lo spettro di energia di particelle con massa ha una gap $2E_0 = 2mc^2$

4- Quando $E \gg E_0$, $E = c(\hbar/2\pi) k$

5- Quando $m=0$ $E \sim k$ per ogni energia. L'elicità (chiralità) è definita

Spettro di energia conico

$$E = (\hbar/2\pi) k v_F$$

Coniugazione di carica
(elettroni \leftrightarrow buche)

$\vec{\sigma} \rightarrow$ Sottoreticolo $A \leftrightarrow B$
(pseudospin)

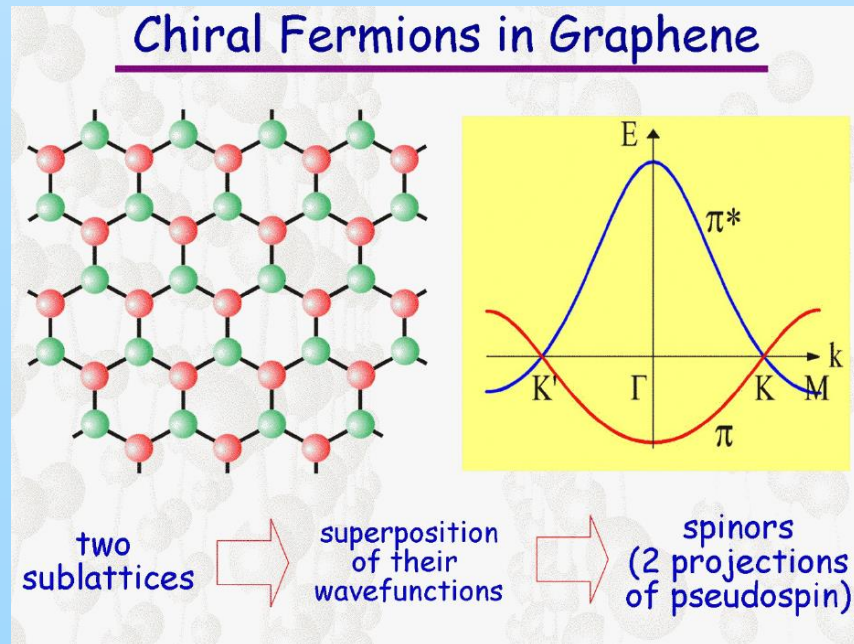


Fermioni di Dirac

Nella materia elettroni e buche si comportano in genere diversamente

Nel grafene mostrano una perfetta simmetria rispettando la regola della simmetria di coniugazione di carica

La funzione d'onda di spinore in QED è rimpiazzata dallo pseudospin σ che identifica il sottoreticolo (A o B) \rightarrow chiralità



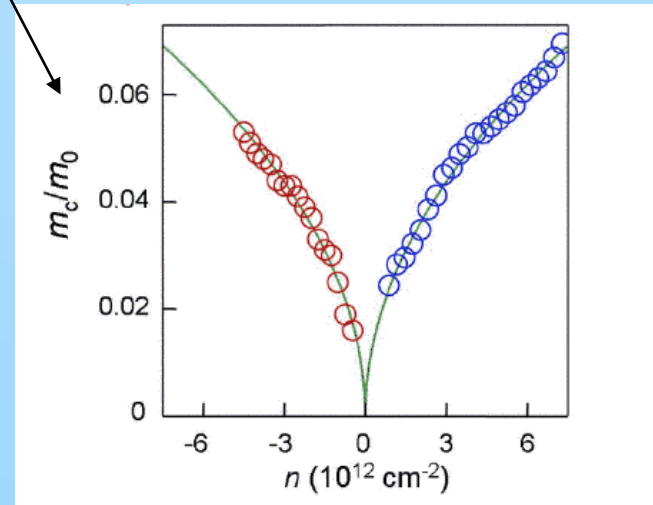
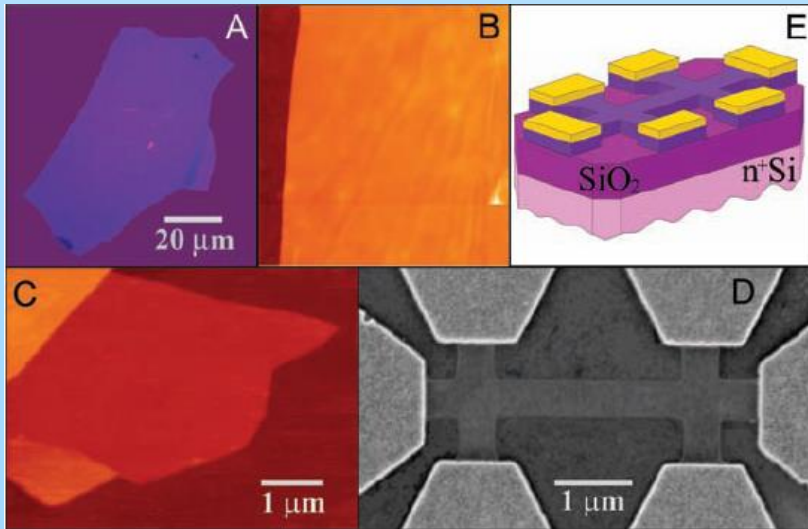
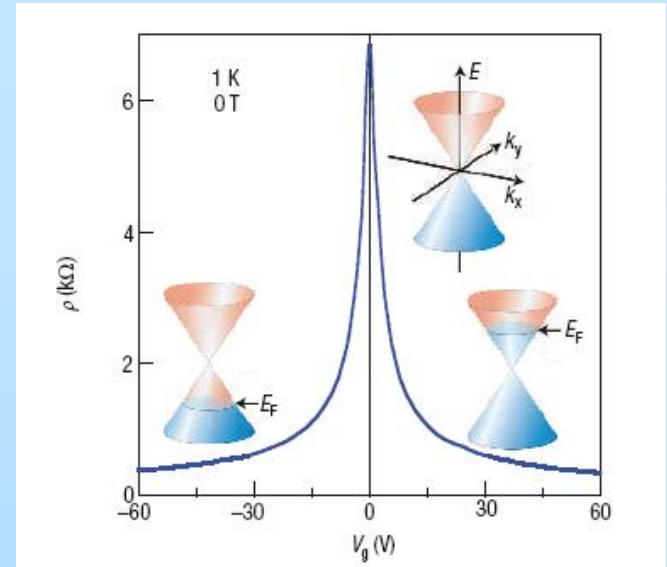


Grafene: Trasporto

$0 < n < 10^{13} \text{ cm}^{-2}$ sia e che h

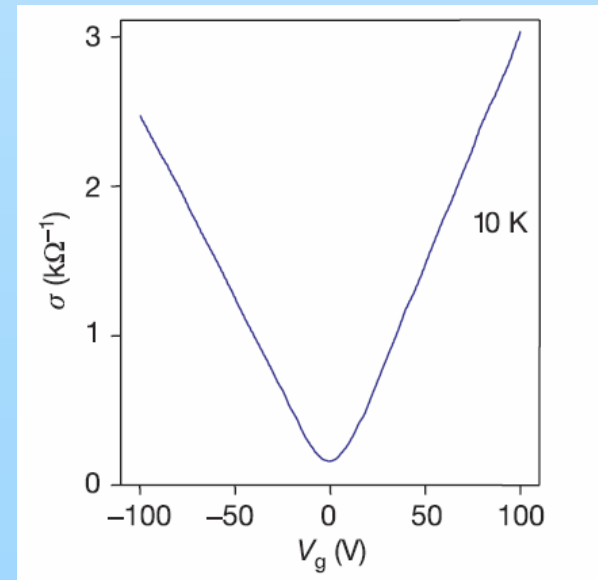
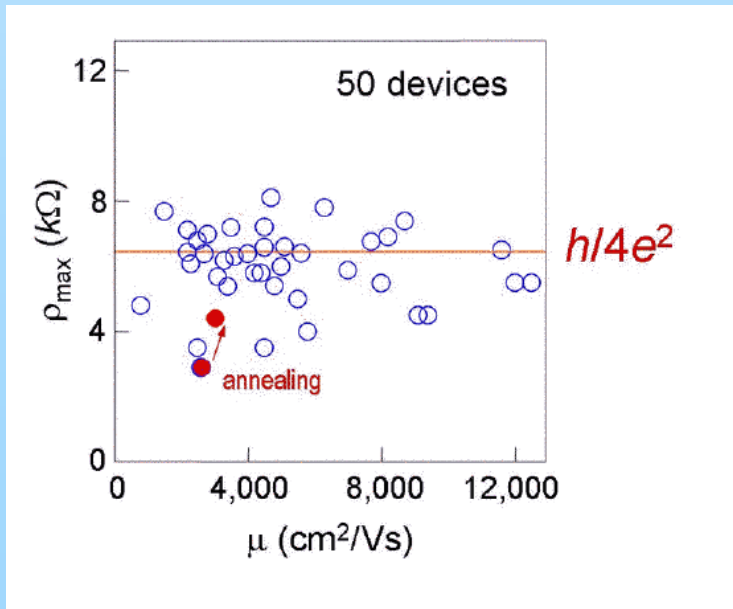
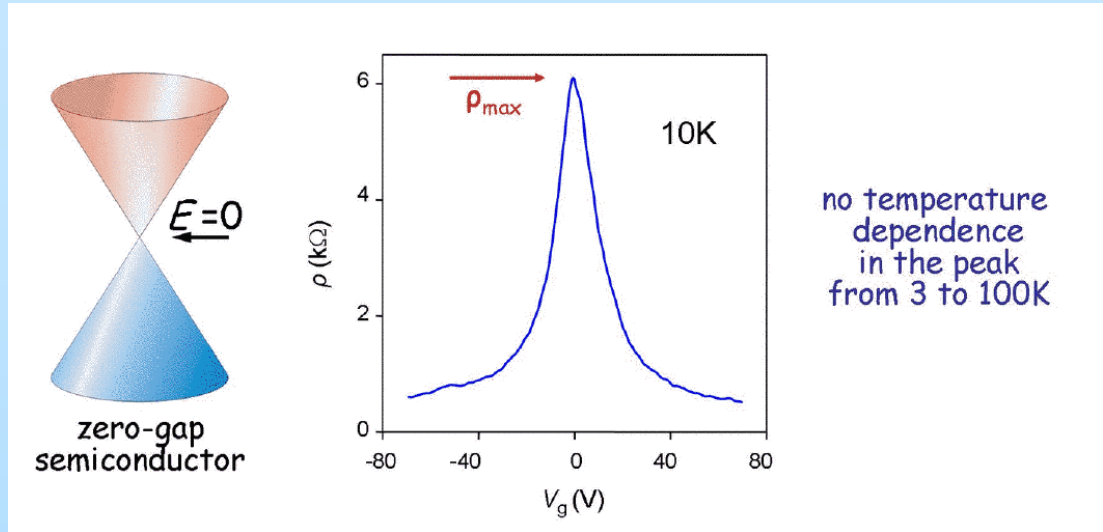
Mobilità: $\mu = \sigma / n$ e $\sim 15000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$

(indip. da T e da n , determinata da impurezze \leftrightarrow semiconduttori)
 Massa ciclotronica m_c
 $v_F = 10^6 \text{ m/s}$





Grafene: Trasporto





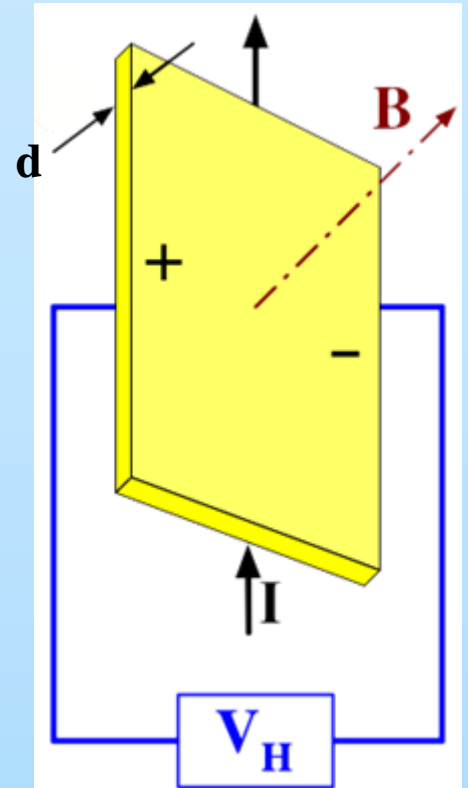
Effetto Hall

$$V_H = \frac{-IB/d}{ne}$$

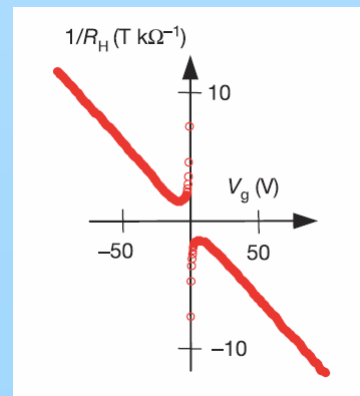
$$R_H = \frac{E_y}{j_x B} = \frac{V_H}{IB/d} = -\frac{1}{ne}$$

$$R_H = \frac{1}{(p - n)e}$$

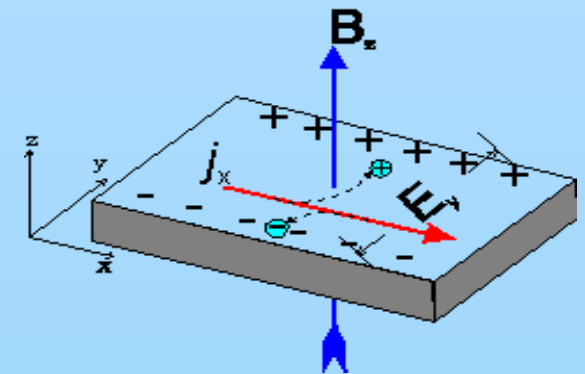
$$\rho_{xy} = \frac{E_y}{j_x} = \frac{B}{ne}$$

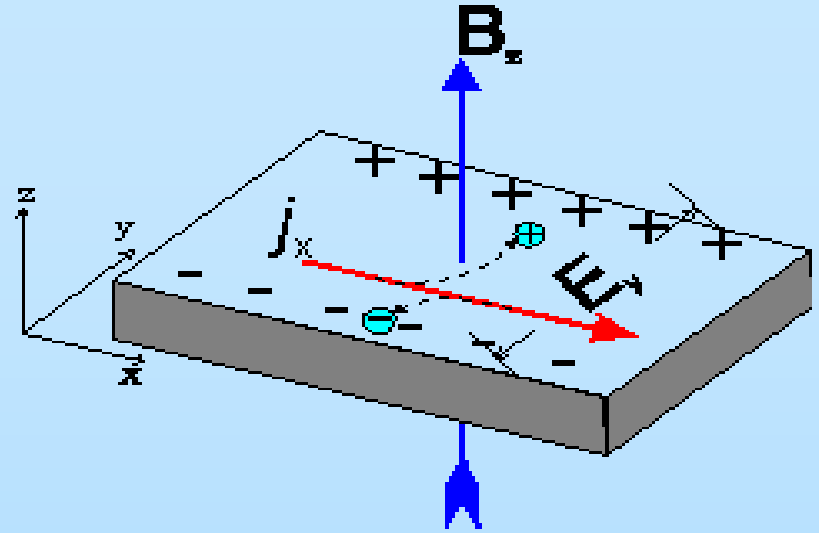


Semiconduttori



Grafene

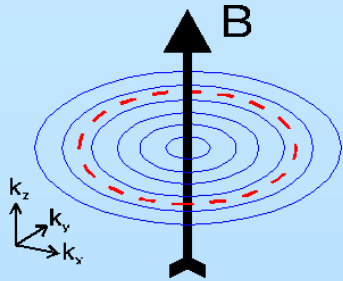






Effetto Hall Quantistico

Nobel Prize 1985 Klaus von Klitzig



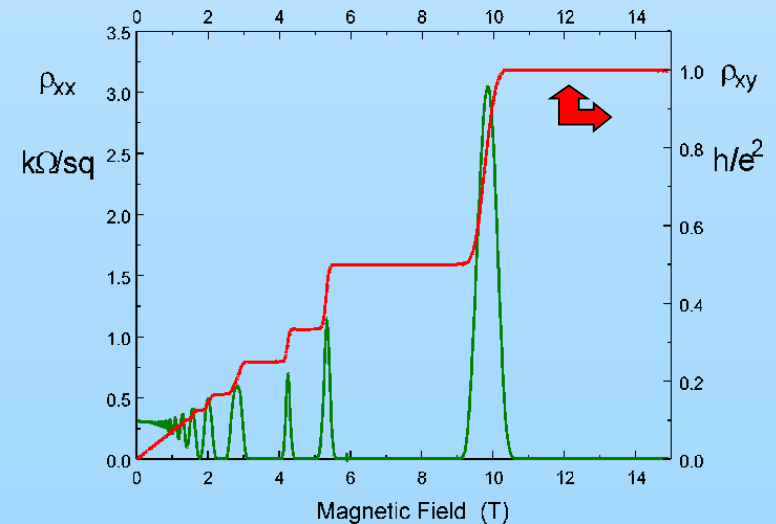
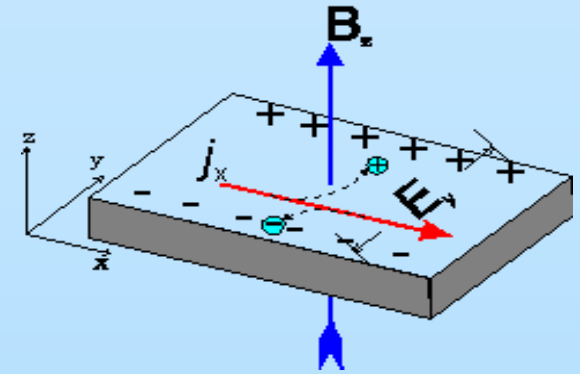
$$\omega_c = \frac{eB}{m^*}$$

$$E = E_i + \left(N + \frac{1}{2}\right) \hbar \omega_c$$

$$n = \frac{eB}{h}$$

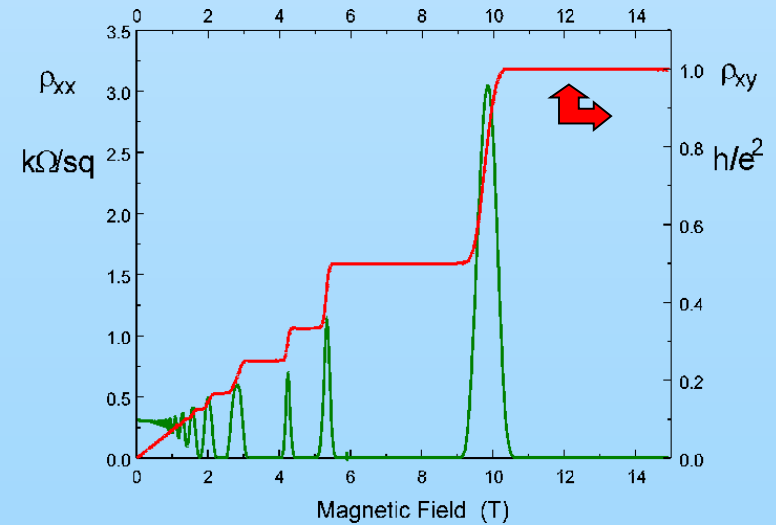
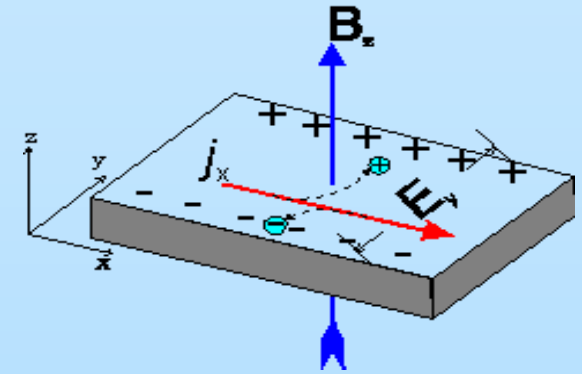
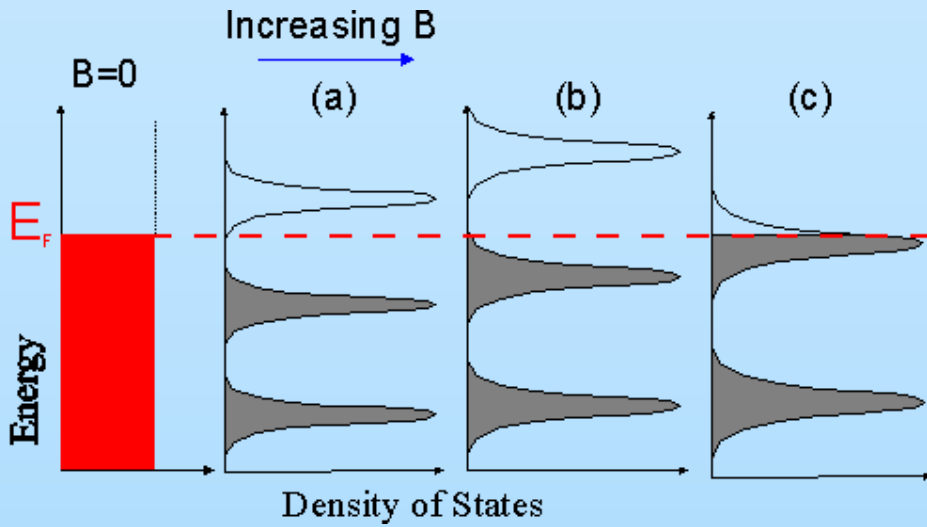
Nr. di stati per ogni livello di Landau

$$\rho_{xy} = \frac{B}{ne} = \frac{h}{ieB} \frac{B}{e} = \frac{1}{i} \frac{h}{e^2}$$





Effetto Hall Quantistico



eterogiunzione GaAs-GaAlAs
a 30mK

$$n = \frac{eB}{h}$$

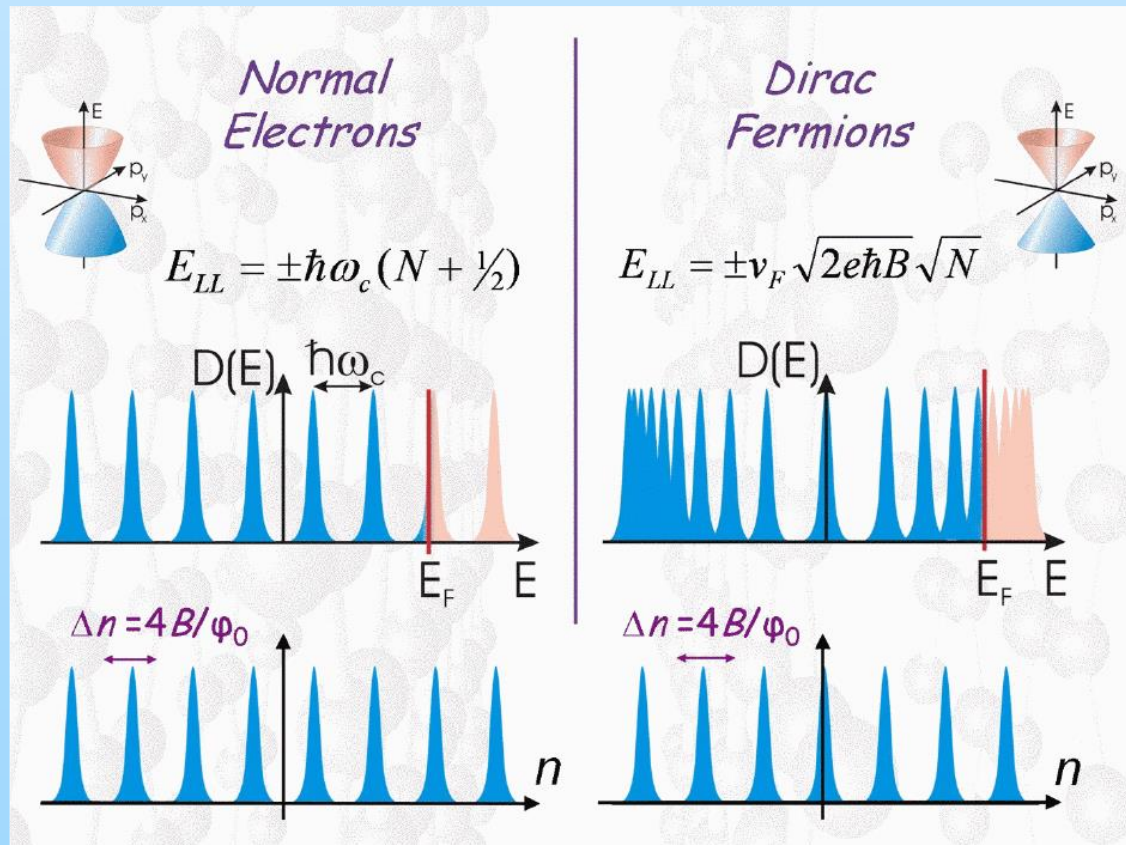
Nr. di stati per ogni
livello di Landau

$$\rho_{xy} = \frac{B}{ne} = \frac{h}{ieB} \frac{B}{e} = \frac{1}{i} \frac{h}{e^2}$$



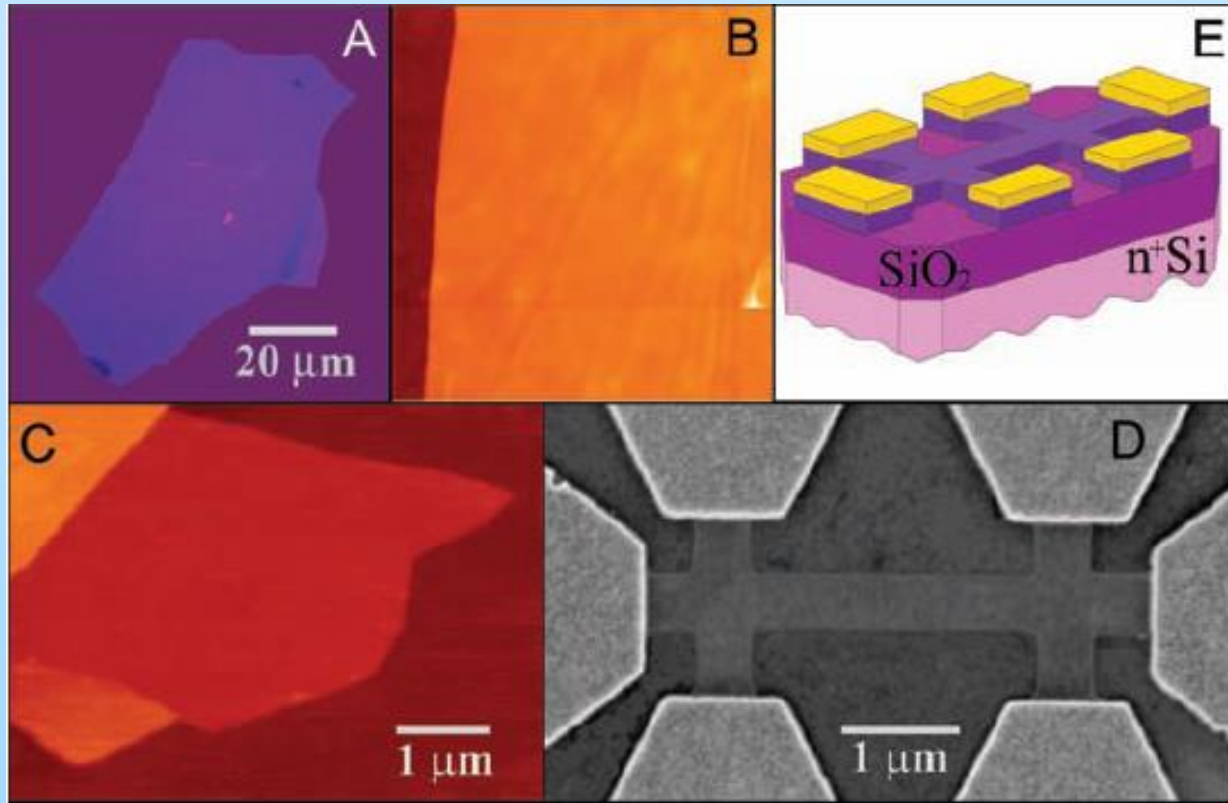
Effetto Hall Quantistico con elettr. di Dirac

Fermioni di Dirac $m=0 \rightarrow E_{v\sigma} = \pm \sqrt{2e\hbar B v_F^2 N}$





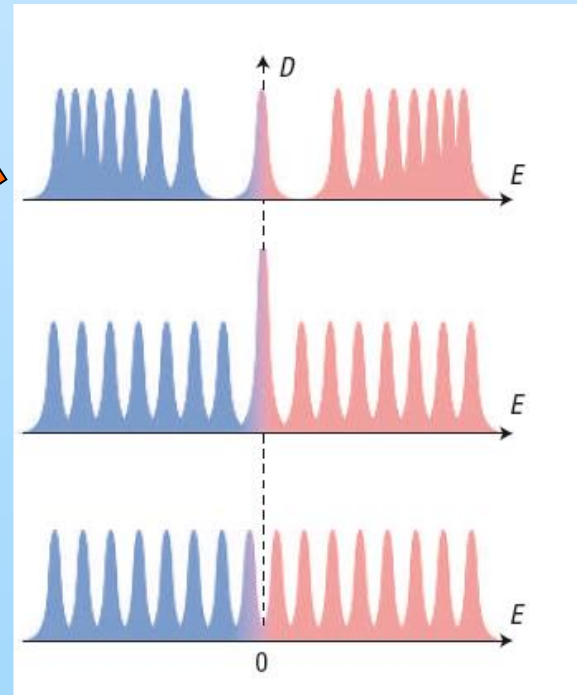
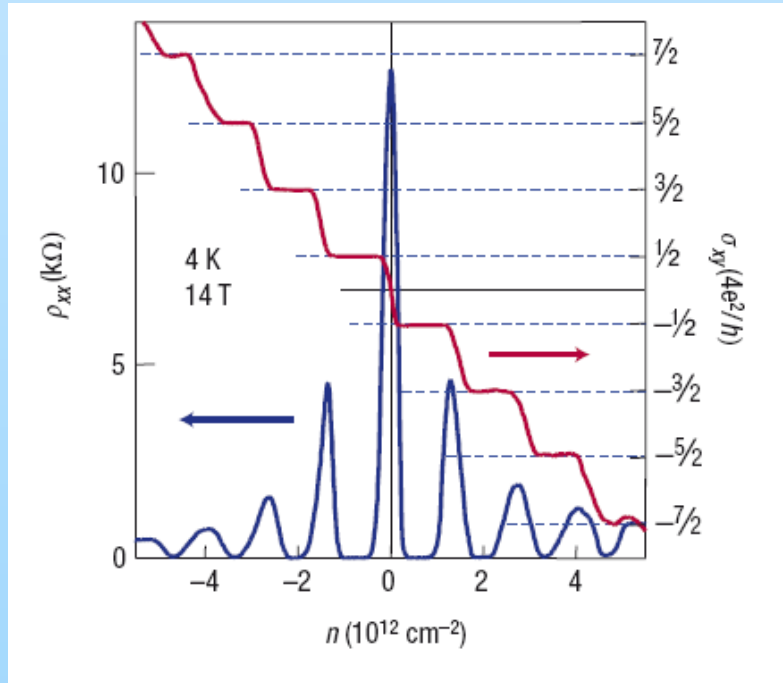
Effetto Hall Quantistico con elettr. di Dirac





Effetto Hall Quantistico Anomalo

Fermioni di Dirac $\rightarrow E_{v\sigma} = \pm \sqrt{2e\hbar B v_F^2 N}$ (QED)



Monolayer

Bilayer A-A
(particella
chirale con
massa:
oximorone)

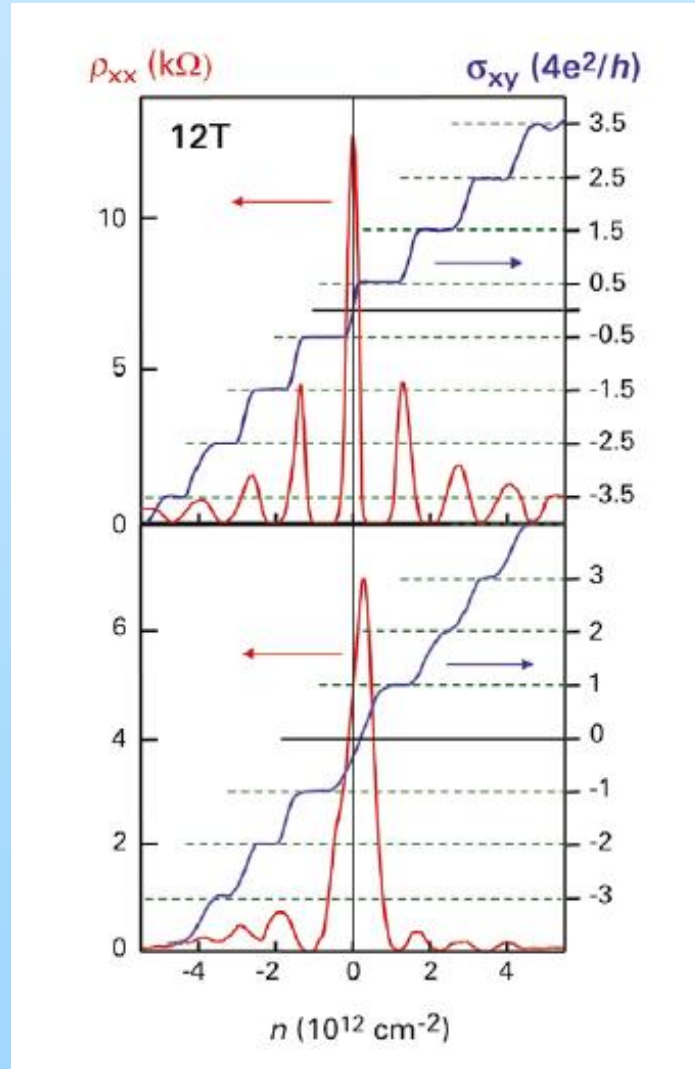
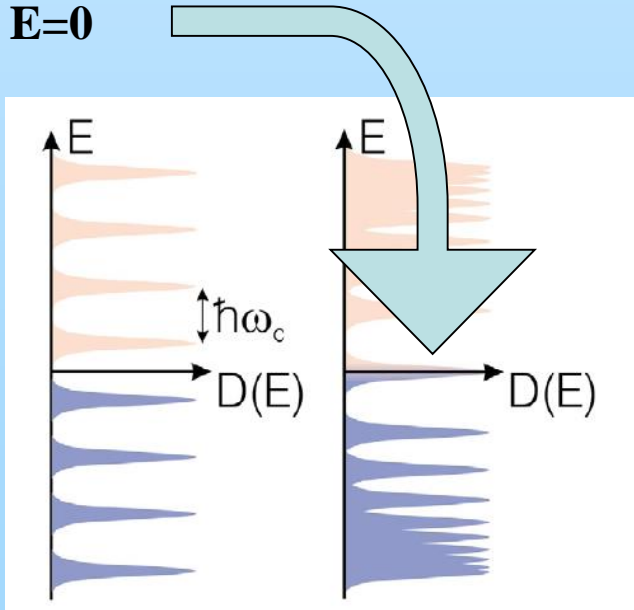
Bilayer A-B
(gap)

K. S. Novoselov et al. Nature 438 (2005) 197



Singolo layer vs. bilayer

L'origine dei plateau quantistici frazionari è nell'esistenza di uno stato ad $E=0$



Singolo

Bilayer A-B

$$E_\nu \sim \sqrt{\nu(\nu-1)}$$

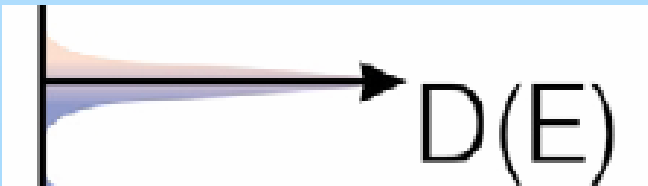
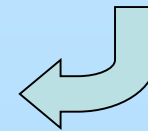


Effetto dei ripples?

Teorema di Atiyah-Singer (T. delle superstringhe) →

Essendo gli stati ad $E=0$ chirali, sono stabili per campi di gauge e curvature dello spazio

I ripple (inomogeneità di B fino a $1T$) **NON** inibiscono l'EHQ anomalo





Quantum Tunneling

Nei semiconduttori la probabilità di tunnel decresce esponenzialmente con l'altezza e la larghezza della barriera (quando $\Delta E < e\hbar$).

Tunnel risonante: quando $E(e) = E(h)$ (nella barriera)

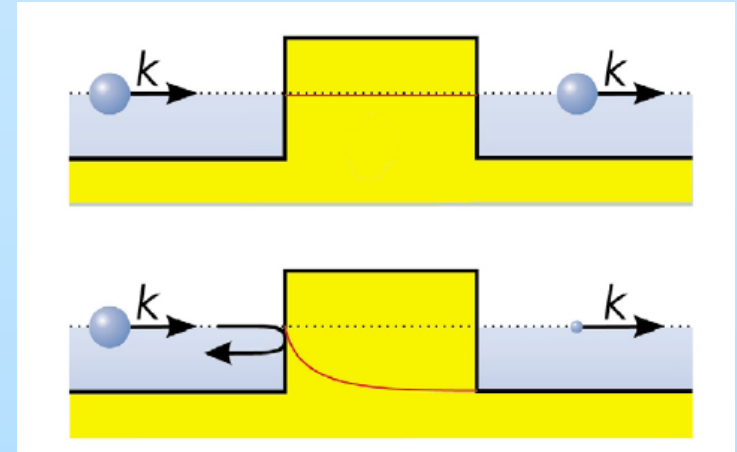
Nel grafene $T=1$

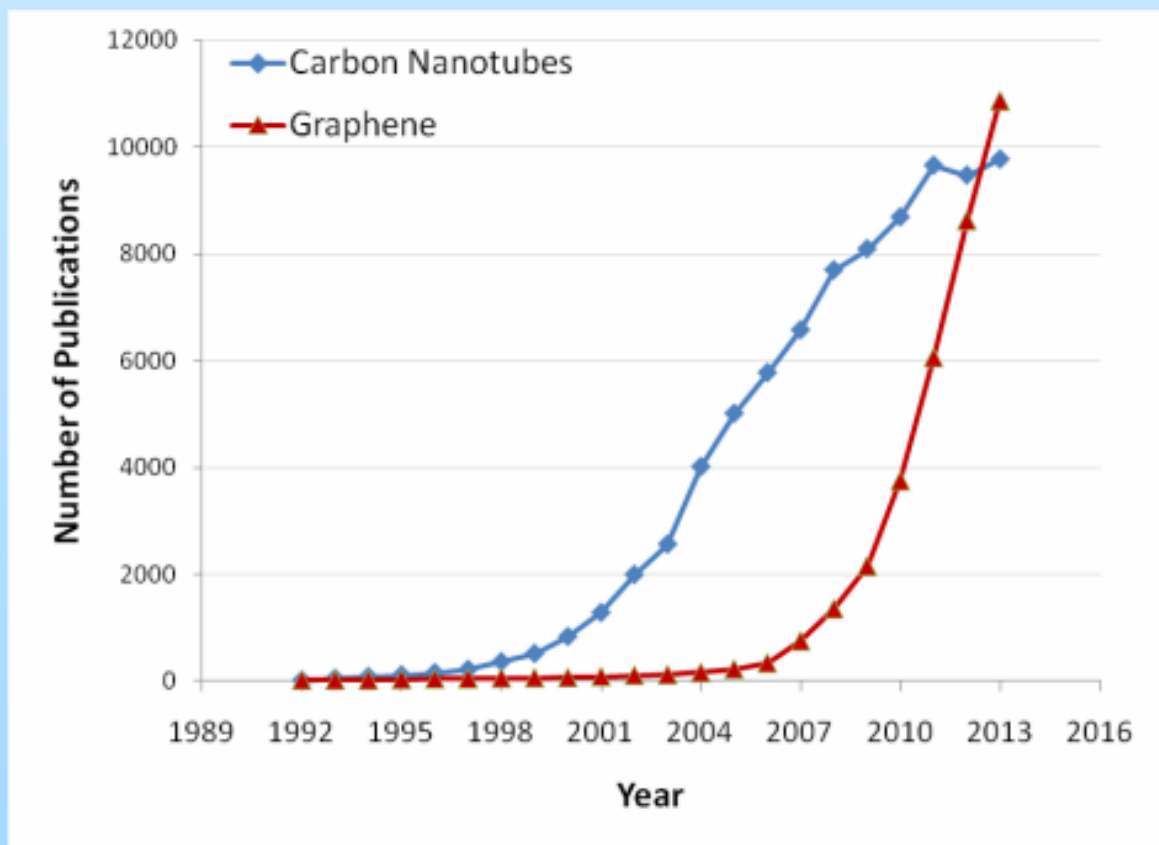


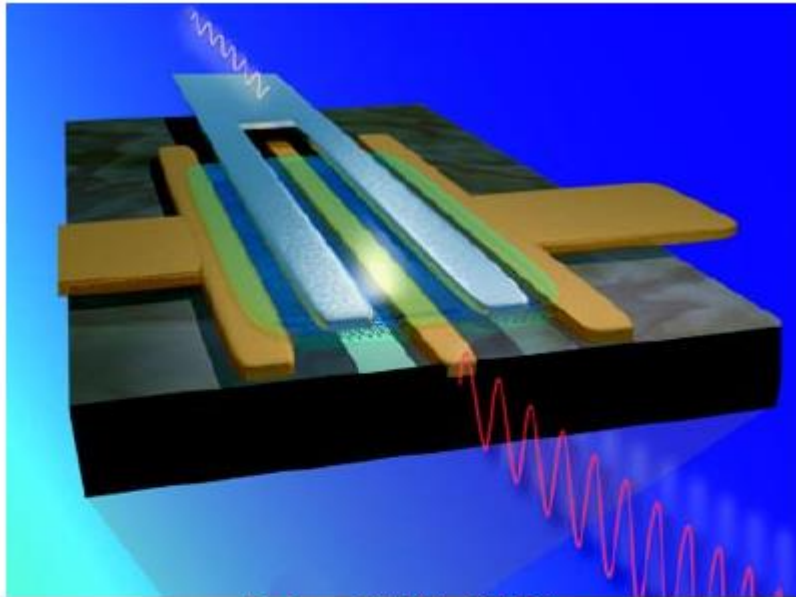
Paradosso di Klein (QED)

Una barriera alta $2m_e c^2$ permette la trasmissione dell'el. attraverso la formazione di una coppia e-p.

Riformulazione del principio di Heisemberg in QED







Graphene FET hits 100 GHz

Physicists in the US have made the fastest graphene transistor ever, with a cut-off frequency of 100 GHz. The device can be further miniaturized and optimized so that it could soon outperform conventional devices made from silicon, says the team. The transistor could find application in microwave communications and imaging systems.

Graphene – a sheet of carbon just one atom thick – shows great promise for use in electronic devices because electrons can move through it at extremely high speeds. This is because they behave like relativistic particles with no rest mass. This, and other unusual physical and mechanical properties, means that the "wonder material" could replace silicon as the electronic material of choice and might be used to make faster transistors than any that exist today.

**Febbraio
2010**

CAPTION: IBM Scientists Demonstrate World's Fastest Graphene Transistor. (PRNewsFoto/IBM)
LOCATION: YORKTOWN HEIGHTS, NY, UNITED STATES
POST DATE: Feb/5/2010 7:59 AM
TAG ID: prnphotos089381
FORMAT: 7.3" x 9.0" @ 300 DPI (2219 x 2700 Color JPEG)
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